Belief identification by proxy*

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Abstract

It is well known that individual beliefs cannot be identified using traditional choice data, unless we exogenously assume state-independent utilities. In this paper, I propose a novel methodology that solves this long-standing identification problem in a simple way. This method relies on extending the state space by introducing a proxy, for which the agent has no stakes conditional on the original state space. The latter allows us to identify the agent's conditional beliefs about the proxy given each state realization, which in turn suffices for indirectly identifying her beliefs about the original state space. This approach is analogous to the one of instrumental variables in econometrics. Similarly to instrumental variables, the appeal of this method comes from the flexibility in selecting a proxy.

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JEL codes: C81, C90, D80, D81, D82, D83.

1. Introduction

Identifying subjective beliefs is a central problem in economics that dates back to the seminal contributions of Ramsey (1931), De Finetti (1937) and Savage (1954). While this literature originally focused on providing foundations for subjective probability, more recently economists have also recognized the practical importance of the question (Manski, 2004). This renewed interest comes from the fact that beliefs are nowadays used for a broad range of purposes, e.g., to make out-of-context predictions; to obtain, compare, and aggregate forecasts; to study irregularities in information processing,

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etc. Therefore, accurately measuring actual beliefs is of utmost importance for applied research and policy.[1]

The traditional methodology for belief identification relies on observing betting behavior: an agent's choices among acts are supposed to reveal her beliefs over the state space (Savage, 1954; Anscombe and Aumann, 1963; Wakker, 1989). However, there is a foundational caveat, known as the *identification problem* (e.g., Drèze, 1961, 1987; Fishburn, 1973; Karni et al., 1983):

> *The agent's beliefs can be identified through traditional choices over acts, only if we exogenously assume her utility function to be state-independent.*

Let me illustrate the problem by means of an example.[2] Suppose that a man suffers from Guillain-Barré syndrom, a serious neurological condition that has left him completely paralyzed, and it is unclear whether he will recover from the disease within the next year. Suppose that we want to identify the subjective probability that his wife attaches to him recovering. Since she loves her husband very much, she values extra money more in a world where he has recovered, compared to a world where he is paralyzed. Hence, we cannot assume her utility over monetary payoffs to be state-independent. As a result, the relationship between her beliefs and her willingness to accept a bet is confounded by her relative utility for money between the two states. And since the latter is no longer normalized to 1 —as it would have been under the state-independence assumption— we cannot identify beliefs from her betting behavior.

This poses a serious problem, as in many economic applications utility functions are state-dependent, either due to intrinsic preferences over states or due to unobservable state-dependent side payoffs. For example, state-dependent preferences commonly arise in insurance problems (Arrow, 1974; Cook and Graham, 1977; Drèze and Rustichini, 2004; Karni, 2008b), legal judgments (Andreoni, 1991; Feddersen and Pesendorfer, 1998; Tsakas, 2017), criminal behavior (Lochner, 2007), real estate decisions (Case et al., 2012), medical decisions (Pauker and Kassirer, 1975, 1980; Lu, 2019), health policy (Delavande, 2008), and in psychology when studying motivated beliefs (Kunda, 1990; Benabou, 2015) or ingroup favoritism (Everett et al., 2015). In all these cases, state-dependent preferences will likely induce errors in belief measurement. Such errors are traditionally overlooked, not because they are not necessarily significant or important, but rather because they cannot be separated from systematic components of the agent's beliefs with the traditional elicitation tools.

Take for instance the extensive literature on motivated beliefs, where the main hypothesis is that intrinsic preferences over the state space lead to systematically biased beliefs in the direction of the preferred state (Kunda, 1990; Benabou, 2015). For instance, people reportedly overestimate their own skill or performance (Svenson, 1981; Eil and Rao, 2011; Zimmermann, 2020; Möbius et al., 2022) or the chances of political events that they like (Bullock et al., 2015; Thaler, 2024). Such overconfidence has important downstream consequences in behavior, e.g., in a political setting, it affects voter turnout and political polarization (Ortoleva and Snowberg, 2015). In order to unbiasedly estimate the effect of overconfidence on behavior, it is important to separate belief distortions due to motivation from possible measurement errors due to falsely assuming state-independent utilities.

Another instance of measurement errors due to state-dependent preferences arises in the surging literature that uses reported beliefs in order to predict behavior in different settings, e.g., in finance (Gennaioli et al., 2015), health (Delavande, 2008) and crime (Lochner, 2007), just to mention a few.

[1] Throughout the paper I assume that an actual belief exists, and it is therefore interpreted as an unobservable primitive. Still, my entire analysis can be easily adapted to classical settings where the belief is interpreted as a parameter that only acquires meaning within a SEU model (Section 6).

[2] This is a modification of the standard example, which was originally used in a letter correspondence between Bob Aumann and Leonard Savage (Drèze, 1987), and has been used widely in this literature since then. We will use this story as our running example throughout the paper. For a formal presentation, see Example 1.

For instance, the subjective probability that women attach to getting pregnant, combined with their choices among birth control methods, allow us to identify their (actual) utilities over pregnancy outcomes (Delavande, 2008), and subsequently to predict women's response to different policy interventions. However, all this presupposes that we elicit women's beliefs without measurement errors, which is rather difficult to do with the usual methods due to the fact that women have obvious stakes in whether they get pregnant or not.

As a final instance of measurement errors because of state-dependent preferences, consider cases where decision makers use experts' expectations as input for their own choices. For instance, small investors will often adopt the forecasts of institutional investors before making their own portfolio allocation choices; occasional gamblers will adopt the forecasts of professional gamblers before deciding where they will place their money; prospective home owner will adopt the forecasts of real estate investors before bidding for a house. However, in practice, all these decision makers rarely take into account that there are usually unobservable state-dependent side payments for these experts, e.g., in the form of an unobservable existing portfolios for the institutional investors or the real estate investors, and similarly in the form of unobservable arbitrage opportunities for the professional gambler (Vlastakis et al., 2009). In the presence of such side payments, the measurement error will likely arise as a consequence of hidden hedging opportunities (Blanco et al., 2010).

Historically, the identification problem was already noticed during the very early days of decision theory, but was consciously put aside: as Leonard Savage admits in his well-known letter correspondence with Bob Aumann, "the problem is serious, but I am willing to live with it until something better comes along" (Drèze, 1987). Most of the proposed solutions rely on non-traditional choice data (see literature overview below). However, none of them is unanimously accepted as the standard one, largely because they require complex and cumbersome data. This suggests that the identification problem is both hard-to-solve and still open.

In this paper, I propose a novel solution, which is both theoretically sound and tractable. The key idea is to stick to the standard methodology of using traditional choice data, albeit over an extended state space. I do this by taking a product space, where one dimension is the original state space and the second dimension is what I call a *proxy*. The crucial feature is that the agent does not have any stakes in the realization of the proxy conditional on our original state space. This allows us to identify her conditional beliefs about the proxy using standard elicitation mechanisms. The main result leverages these conditional beliefs about the proxy to uniquely identify the actual belief about the original state space (Theorem 1).

Let me illustrate the identification result in the context of the running example. Suppose that there is a promising new experimental drug in the market which is believed to expedite the recovery time from Guillain-Barré. The husband is eligible to participate in the last phase of the clinical trial. A proxy describes the two possible contingencies regarding his participation, i.e., he will receive either the drug (t_1) or a placebo (t_2). The wife is informed that his chances of being placed in the treatment group are 50%, but she is not told in which group he is actually placed in the end. Of course, if she were to learn that he received the drug, her belief would change to some $\nu := \pi_S(\cdot|t_1)$; on the other hand, if she learned that he received the placebo, her belief would remain unchanged at $\mu := \pi_S(\cdot|t_2)$.[3] For a graphical illustration, see Figure 1 below.

The crucial feature is that the wife *does not have any stakes in the realization of the proxy conditional on her husband's health status.* For instance, conditional on him recovering (resp., remaining paralyzed), she will not care about whether he has taken the drug or the placebo, and therefore her

[3]Here, I implicitly assume that the husband does not even know that he is participating in the clinical trial, and therefore his health cannot be affected by psychological factors, such as the well-known placebo effect.

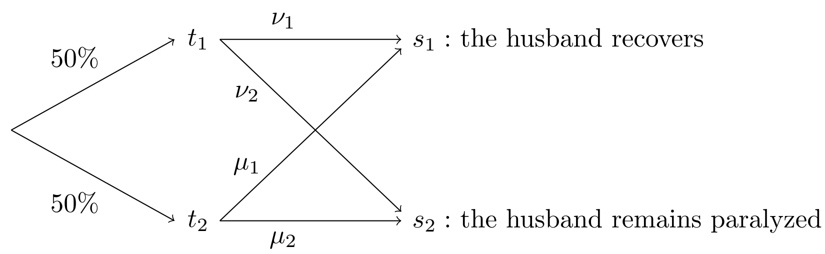

Figure 1: The two contingencies that the proxy describes are "the husband takes the experimental drug (t_1)" and "the husband takes the placebo (t_2)". The prior probability of the husband (randomly) being placed in the control group is 50%. Conditional on the placebo being taken, the wife's belief remains at μ. Conditional on the drug being taken, her belief changes to ν.

utility from money will not depend on the realization of the proxy.[4] Hence, we can identify her conditional beliefs over T given that her husband has recovered (viz., $\pi_T(\cdot|s_1)$), as well her conditional beliefs over T given that he has remained paralyzed (viz., $\pi_T(\cdot|s_2)$). In both cases, we can do this, using standard elicitation tasks, *without worrying about the identification problem* (Section 4).

Then, as long as S and T are not independent, we can uniquely identify her conditional belief $\pi_S(\cdot|t_2)$ given that he has taken the placebo, which is the belief μ she would have held if we had not introduced the proxy. A generalization of this simple example to any finite state space and any suitable proxy constitutes my main identification result (Theorem 1).

Notably, my identification approach bears a striking conceptual similarity with the use of instrumental variables in econometrics. Recall that in IV regression we replace one awkward exogenous assumption (viz., orthogonality) with another exogenous assumption which is easier to justify (viz., the exclusion restriction). In the same way, instead of exogenously assuming that the wife does not care about her husband's health status, we assume that she does not care about his treatment conditional on his eventual health status, with the latter being much easier to defend than the former. Thus, in both settings, it is not that we stop imposing exogenous assumptions all together, but rather that we strategically select the domain (viz., the instrumental variable and the proxy, respectively) so that the exogenous assumptions can be justified on the basis of common sense and/or existing literature.

The flexibility to choose the instrumental variable is what has made IV regression so popular. Likewise, the flexibility in choosing a proxy from a large pool of potential candidates, is what makes belief identification by proxy an appealing method. In Section 7, I show that it is indeed essentially always possible to construct a suitable proxy, even in cases where we cannot easily pick one off the shelf. Then, I go on to revisit some applications where identification by proxy corrects the belief measurement error which is likely to arise due to falsely assuming state-independent utilities.

The existing literature is roughly split in two streams, one that aims at providing tools for belief elicitation purposes, and an axiomatic one that focuses on achieving identification theoretically. A major conceptual difference between these two streams is that the former explicitly assumes that beliefs are an actual primitive (similarly to this paper), whereas the latter treats beliefs as a parameter within a model without much concern on whether these are actual beliefs or not. For an overview, see Drèze and Rustichini (2004), Grant and van Zandt (2008), Karni (2008a, 2014) and Baccelli (2017).

Starting with the first stream, the only papers that introduce mechanisms for eliciting beliefs under state-dependent preferences are Karni (1999) and Jaffray and Karni (1999), with the latter

[4]The implicit assumption that I make for the sake of this simple example is that the drug is known not have any side-effects that would make the wife care about whether her husband has taken it or not conditional on eventual health condition.

proposing two different mechanisms. In particular, Karni (1999) and the first mechanism of Jaffray and Karni (1999) rely on assuming bounded state utilities, and they approximate the actual beliefs as monetary incentives grow arbitrarily large. The problem is that this approach requires either extreme experimental costs, or hypothetical data. These pitfalls are recognized by the authors, who point out that in those early days there was no other option (e.g., Karni, 1999, p.485). The second mechanism in Jaffray and Karni (1999) assumes state-dependence in the form of unobserved state-dependent side payments: first it proceeds to elicit these payments, and subsequently to elicit beliefs using standard techniques. Unfortunately, this is a rather restrictive setting: in many applications, preferences over states are intrinsic. Moreover, eliciting the state-dependent payments is data-demanding.

Turning to the second stream, the various attempts within axiomatic decision theory differ in terms of the non-traditional choice domain they consider. There are two main methodological approaches, both relying on the agent's beliefs being somehow revised at some instance.

The first such method, which was mainly followed in the early days, is based on exogenously manipulating the context and a fortiori the agent's beliefs. For instance, Fishburn (1973) allows for comparison between acts conditional on different events. Karni et al. (1983), Hammond (1999) and Karni and Schmeidler (2016) introduce hypothetical preferences over acts conditional on exogenously given probabilities over the states. Schervish et al. (1990) allow the agent to compare lotteries at different states. Karni (1992, 1993a) allows the analyst to observe preferences conditional on different events. In some way, all of these papers assume the analyst to observe the agent's hypothetical choices. This feature makes the implementation of these methods cumbersome.

The second method relies on the idea that the agent herself can affect the state realization, and is called the *moral hazard approach*. It originally appeared in the early sixties (Drèze, 1961), before resurfacing (Drèze, 1987; Drèze and Rustichini, 1999) and recently receiving attention again (Baccelli, 2021). This literature relates to my work on a high level. The difference is that in moral hazard the analyst relies on the agent influencing the state realization, whereas in my case it is sometimes the analyst who may choose to influence the state realization (Example 4). The second major difference is that the moral hazard methodology applies only to settings where the state has not been realized yet, while my method also applies to factual beliefs.

Closer to my paper is the recent work of Lu (2019), which leverages stochastic choice data in order to identify the agent's beliefs. This approach relies on observing choices conditional on (many) independent realizations of a given signal. It is similar in spirit to mine, in that it also extends the state space by considering a second dimension (i.e., the set of signal realizations). Its main drawback is that it requires a large dataset of independent observations, which is not always feasible, e.g., stochastic choice data cannot be collected when uncertainty is about a one-shot event, like for instance the husband's health in our running example, or the outcome of the upcoming presidential elections. I further elaborate on the relationship between my work and the one of Lu (2019) in Section 3.

Between the moral hazard approach and the aforementioned stochastic choice approach, one can place a sequence of papers that rely on the agent influencing the state realization and choosing an act, conditional on different signals (Karni, 2011a,b, 2014). The common element across these papers of Karni, the paper of Lu, and my work is that all can be thought to rely on traditional choice data over an extended state space. Nevertheless, the way this general idea is implemented is very different.

There is also related work on the identification problem within non-expected utility models: While the problem is resolved in some cases outside SEU (Chew and Wang, 2020; Mononen, 2023), in other cases it persists (Karni, 2020).

Finally, my work is methodologically related to a recent paper by Alaoui and Penta (2024), whose aim is to separately identify risk preferences from utility from wealth. Their identification strategy relies on introducing an additional dimension in the form of a yardstick, which plays a similar role to the one that the proxy plays in my paper.

Section 2 presents the background and formally introduces the identification problem. Section 3 presents my method and formally states my main identification result. Section 4 relates my approach with the standard belief elicitation mechanisms. Section 5 leverages my identification result to provide a well-founded definition of actual utility function. Section 6 provides decision-theoretic foundations for a proxy, thus formally distinguishing between exogenous and testable assumptions. Section 7 elaborates into practical aspects of my approach. Section 8 concludes. All proofs are relegated to the Appendix.

2. The identification problem formalized

Consider a finite state space $S = \{s_1, \dots, s_K\}$. A (female) agent has a full-support belief $\mu \in \Delta(S)$, called the *actual belief*, which we want to identify.

An act is a function $f : S \to Q$ that maps each state $s \in S$ to a consequence $f_s := f(s)$ in a convex subset Q of a finitely dimensional Euclidean space. The set of all acts is denoted by $\mathcal{F}_S$. Observed choices among acts are henceforth called traditional choice data.[5] Suppose that the agent's choices are consistent with a preference relation $\trianglerighteq$ over $\mathcal{F}_S$ that admits a *State-Dependent Subjective Expected Utility (abbrev., SEU)* representation. That is, there exists some state-dependent utility function $u : S \times Q \to \mathbb{R}$ such that, the preference relation $\trianglerighteq$ is represented by the SEU function

$$\mathbb{E}_\mu(u(f)) := \sum_{s \in S} \mu(s) u_s(f_s), \tag{1}$$

where $u_s(q) := u(s, q)$ is the state-utility from outcome q at state s. Formally, this means that for any two acts $f, g \in \mathcal{F}_S$, the following equivalence holds:

$$f \trianglerighteq g \;\Leftrightarrow\; \mathbb{E}_\mu(u(f)) \geq \mathbb{E}_\mu(u(g)). \tag{2}$$

Whenever this is the case, we will say that the pair (u, μ) represents $\trianglerighteq$.

The appeal of SEU is that it allows us to separate beliefs from utilities. Is this enough for identifying the agent's actual beliefs? Unfortunately, not! The reason is that the pair (u, μ) is not the only SEU representation. Namely, take any other full-support belief $\tilde{\mu} \in \Delta(S)$, and define the rescaled utility function $\tilde{u} : S \times Q \to \mathbb{R}$ so that, for every $s \in S$, it is the case that

$$\tilde{u}_s = \alpha_s + \beta \frac{\mu(s)}{\tilde{\mu}(s)} u_s, \tag{3}$$

where $a_s \in \mathbb{R}$ and $\beta > 0$. Then, for every $f \in \mathcal{F}_S$, we obtain

$$\mathbb{E}_{\tilde{\mu}}(\tilde{u}(f)) = \alpha + \beta \mathbb{E}_\mu(u(f)), \tag{4}$$

where $\alpha := \sum_{s \in S} \alpha_s \tilde{\mu}(s)$. Hence, the pair $(\tilde{u}, \tilde{\mu})$ constitutes an alternative SEU representation of the same preferences. So, even if we had access to the complete preference relation (e.g., by observing data from a complete experiment), we would not be able to tell if the actual belief is μ or $\tilde{\mu}$, i.e., beliefs cannot be identified from choices over $\mathcal{F}_S$. This is known as the *identification problem* of SEU.

[5] Formally, traditional choice data take the form of a collection of menus $F \subseteq \mathcal{F}_S$ (i.e., the experiment), together with a choice correspondence that associates each menu F in this experiment with a non-empty set of observed choices $C(F) \subseteq F$ (i.e., the data). The experiment is called complete whenever it contains every compact subset of $\mathcal{F}_S$.

The early solution to the identification problem was to essentially assume it away. This approach boils down to selecting a *State-Independent Subjective Expected Utility (abbrev., SI-SEU)* representation $(\bar{u}, \bar{\mu})$ among the infinitely many SEU representations, i.e., one such that $\bar{u}_s(q) = \bar{u}(q)$ for all $s \in S$ and all $q \in Q$. It is well-known that all SI-SEU representations involve the same belief $\bar{\mu}$ (Savage, 1954; Anscombe and Aumann, 1963; Wakker, 1989). Then, according to this early solution, this belief $\bar{\mu}$ is labelled as the actual belief.[6] This is known as the *assumption of state-independent utilities*. Unfortunately, as illustrated in the following example, this assumption corresponds to an arbitrary model selection.

Example 1. (The wife's problem). Recall the introductory example, in which a man suffers from Guillain-Barré syndrom, and it is unclear whether he will recover (state s_1) or remain paralyzed (state s_2). Suppose that his wife is a risk-neutral SEU maximizer, i.e., she evaluates monetary payoffs, at states s_1 and s_2 respectively, using the state-utility functions

$$u_1(q) = \gamma_1 q \text{ and } u_2(q) = \gamma_2 q,$$

with $\gamma_1, \gamma_2 > 0$. So, when we observe her being indifferent between buying and not buying an insurance with payout of \$100k and premium of \$10k, we conclude that her beliefs satisfy

$$\frac{\mu(s_2)}{\mu(s_1)} = \frac{\gamma_1}{9\gamma_2}. \tag{5}$$

Obviously, in order to identify her actual belief, we must specify the parameters γ_1 and γ_2. State-independent utilities correspond to one such specification, which sets $\gamma_1 = \gamma_2$, and subsequently yields $\bar{\mu}(s_1) = 0.90$. However, this is just one of the infinitely many parameter specifications that satisfy Equation (5), and in this sense it is an arbitrary choice. ◁

To complicate things even further, oftentimes a SI-SEU representation does not even exist in the first place. This is for instance the case when state-preferences over consequences differ across states, either in some inherent way (Example 2), or because of unobservable state-contingent side payments (Example 3).

Example 2. Continuing with the same example, suppose that the wife's preferences were instead represented by the SEU representation (u, μ), where

$$u_1(q) = q^2 \text{ and } u_2(q) = q.$$

The idea is that the wife is risk-seeking whenever her husband is healthy. In order for a SI-SEU representation to exist, it must necessarily be the case that $u_1 = \alpha + \beta u_2$ for some $\alpha \in \mathbb{R}$ and $\beta > 0$. It is not difficult to verify that such parameters do not exist. ◁

Example 3. In the running example, suppose that the wife's preferences were instead represented by the SEU representation (u, μ), where

$$u_1(q) = q^2 \text{ and } u_2(q) = (c + q)^2.$$

The interpretation is that the wife is risk-seeking with quadratic state-utility function at both states, and has an unobservable side payoff at s_2, e.g., the wife has already bought another insurance that we are not aware of. Once again, there are no parameters $\alpha \in \mathbb{R}$ and $\beta > 0$ such that $u_1 = \alpha + \beta u_2$. So again, there is no SI-SEU representation. ◁

[6] Strictly speaking, the same belief $\bar{\mu}$ would have been uniquely identified even if we had started with a quasi state-independent utility function, $\bar{u}_s(q) = \alpha_s + \bar{u}(q)$, instead. The crucial property of such a representation is that the state-independent and the state-dependent part of the utility function are linearly separable. This is because, by (3) and (4), adding a state-dependent constant does not affect the belief in the SEU representation. I thank an anonymous referee for pointing out this subtle issue. I further elaborate in Section 4.

Whenever there is no SI-SEU representation, the identification problem becomes immediately visible, as there is no obvious candidate among the infinitely many SEU representations to exogenously assume. However, this does not mean that in the converse scenario where a SI-SEU representation exists, the identification problem is resolved. In fact, the existence of a SI-SEU representation and the identification problem are conceptually orthogonal: the former is about axioms of choice being satisfied, while the latter is about model specification, conditional on these axioms being satisfied. So, why is there a widespread misconception that they are linked? The reason is twofold and can be traced back into the history of SEU.

First of all, in the early days of decision theory, the literature was not interested in identifying actual beliefs. Instead, the focus was on predicting choices among acts in $\mathcal{F}_S$, and on establishing that there exists a well-founded definition of subjective probability. And since SI-SEU representations served both purposes, the theoretical literature started treating $(\bar{u}, \bar{\mu})$ as the default model, whenever it existed. Thus, when the applied literature eventually started being interested in identifying actual beliefs, it simply adopted $\bar{\mu}$ as 'the correct definition of actual belief'. This is highlighted in two facts: first, (almost) the entire belief elicitation literature consists of mechanisms that elicit $\bar{\mu}$ (see Section 4 for an overview of this literature); and second, the entire literature on State-Dependent SEU is motivated by the non-existence of SI-SEU representation (Drèze and Rustichini, 2004; Grant and van Zandt, 2008; Karni, 2008a, 2014; Baccelli, 2017, and references therein).

The second source of the misconception comes from the fact that state-independent utilities are often confused terminologically with state-independent preferences, with the latter being typically captured by axioms such as the state-monotonicity axiom in Anscombe and Aumann (1963), or P3-P4 in Savage (1954), or state-independent preference intensity in Wakker (1989). However, state-independent preferences are weaker than state-independent utilities, as the former simply guarantees the existence of a SI-SEU representation, whereas the latter selects a SI-SEU representation as the 'correct model' whenever one exists.

But how big of a problem is it to assume state-independent utilities (assuming of course that a SI-SEU representation exists)? The answer depends on the purpose of belief identification. If we simply want to predict the wife's insurance choices in Example 1, it is no big deal to assume state-independent utilities, as all SEU models (including SI-SEU) deliver the exact same predictions. If, on the other hand, we want to use the wife's beliefs for other purposes (e.g., as data in an experiment that studies motivated beliefs, or for predicting out-of-context if she will book a vacation for next summer), we need to select a SEU representation that involves her actual belief. In this last case, it is reasonable to assume state-independent utilities only if we are confident that *the agent has no stakes* over the state realization. We further elaborate on this point in Section 7.2, where we present several economic applications where the literature naively assumes state-independent utilities.

Remark 1. Regarding terminology, throughout the paper, *"having no stakes in the state realization"* is used synonymously to *"the utility function being state-independent"*. As a side remark, Ramsey (1931) referred to this case as one where all relevant events in S are *"ethically neutral"*. $\triangleleft$

3. Main identification result

In this paper, I propose a novel approach that solves the identification problem in a theoretically sound and practically tractable way. The idea is to continue using traditional choice data, albeit over an appropriately extended state space. Formally, I introduce another state space $T = \{t_1, \ldots, t_N\}$, and define the product space $S \times T$. The agent's actual belief over the extended state space is denoted by $\pi \in \Delta(S \times T)$. For any nonempty $A \subseteq S$ and $E \subseteq T$, we respectively denote the marginal conditional beliefs $\pi_T(E|A) := \pi(A \times E|A \times T)$ and $\pi_S(A|E) := \pi(A \times E|S \times E)$. Marginal (unconditional) beliefs are simply denoted by π_T and π_S respectively.

Definition 1. (Proxy). We say that T is a proxy for S whenever the following are satisfied:

(P_0) Conditional no stakes: The agent has no stakes in T conditional on the realization of S.

(P_1) Uninformative event: There is some event $E \subseteq T$ such that $\pi_S(\cdot|E) = \mu$.

(P_2) Objective marginal belief: π_T agrees with an objective probability measure π_T^{obj}.

(P_3) Linear independence: $\pi_T(\cdot|s_1), \dots, \pi_T(\cdot|s_K)$ are linearly independent in $\mathbb{R}^T$.

Condition (P_0) is the key feature of T. Formally, this is satisfied if and only if any incentive-compatible mechanism will elicit the agent's actual conditional beliefs $\pi_T(\cdot|s)$ for all $s \in S$ via the strategy method (see Section 4). Of course, ex ante the agent may still care about the realization of T, due to the correlation with S. But once uncertainty about S has been resolved, she has no longer any stakes in the realization of the proxy.

Condition (P_1) states that conditioning with respect to $E \subseteq T$ does not provide any information about S in comparison to the benchmark case where T is not introduced in the first place. Thus, E is called the uninformative event. Notice that the event E is not necessarily equal to T itself, meaning that the actual belief μ does not necessarily coincide with the unconditional marginal π_S (Examples 4 and 6 below).

Condition (P_2) consists of two parts. First, uncertainty about T is described by an objective marginal belief, π_T^{obj}, which is formed based on commonly known facts; second, it is assumed that the agent's actual marginal belief π_T agrees with the objective belief. This assumption often reflects the idea that π_T^{obj} has been publicly announced by the analyst (Examples 4 and 6 below), while in other cases it describes some demographic characteristic whose distribution is already commonly known to be π_T^{obj} (Example 5 below).

Condition (P_3) is perhaps the least-obvious of the four conditions. Loosely speaking, it guarantees that there is no redundant information within T. Whenever S is binary, this condition reduces to $\pi_T(\cdot|s_1) \neq \pi_T(\cdot|s_2)$, i.e. S and T are not independent.

Let me provide some examples for different types of proxies, while also illustrating the four conditions of Definition 1.

Example 4. (New drug). Recall the wife's problem, and similarly to the introduction suppose that there is a promising new experimental drug which supposedly helps recovery. The husband is eligible to participate in a clinical trial. The proxy describes the two possible groups in which the husband can be placed:

$$T = \{\text{treatment group } (t_1),\ \text{control group } (t_2)\}.$$

Then, I argue that $(P_0) - (P_3)$ satisfy common sense:

- P_0 : The wife will not care if he has taken the drug or the placebo, once the husband's health condition is known.
- P_1 : The wife believes that in case he receives the placebo, his chance of recovery will remain unchanged, i.e., $\pi_S(\cdot|t_2) = \mu$. Hence, $\{t_2\}$ is the uninformative event.
- P_2 : The wife knows that the chance of him being included in the treatment group is 50%.
- P_3 : The wife believes that the drug will affect his recovery probability, i.e., $\pi_T(\cdot|s_1) \neq \pi_T(\cdot|s_2)$.

Such proxies are called *influential actions* (Tsakas, 2020), as the analyst (stochastically) influences the state realization via the proxy. ◁

Example 5. (Relevant gene). We consider the wife's problem once again. It is commonly known that recovery is correlated with the presence of a specific gene that 50% of all males carry. The gene has no other consequence. The wife does not know if her husband carries it or not. So, the proxy becomes:

$$T = \{\text{gene } (t_1), \text{ no gene } (t_2)\}.$$

Once again, I argue that $(P_0) - (P_3)$ satisfy common sense:

P_0 : Conditional on the husband's health condition, the wife does not care if he has the gene.

P_1 : Her actual belief is the one she holds before learning whether he has the gene or not, i.e., $\mu = \pi_T(t_1)\pi_S(\cdot|t_1) + \pi_T(t_2)\pi_S(\cdot|t_2)$. Hence, T is the uninformative event.

P_2 : The wife knows that the probability of the husband having the gene is 50%.

P_3 : The wife knows that recovery is correlated with the gene, i.e., $\pi_T(\cdot|s_1) \neq \pi_T(\cdot|s_2)$.

Such proxies are induced by a *demographic characteristic* whose distribution is commonly known. ◁

Example 6. (Expert versus charlatan). We consider the wife's problem for a third time, supposing now that the wife is told that her husband's file was examined by some doctor who subsequently predicted that he will recover. The proxy describes the two possible expertise levels of the doctor:

$$T = \{\text{expert } (t_1), \text{ charlatan } (t_2)\}.$$

Then again, I argue that $(P_0) - (P_3)$ are reasonable conditions:

P_0 : Conditional on the husband's health condition, the wife will not care if the diagnosis has come from the expert or the charlatan.

P_1 : Conditional on the diagnosis coming from a charlatan, the wife does not revise her beliefs, i.e., $\pi_S(\cdot|t_2) = \mu$. Hence, $\{t_2\}$ is the uninformative event.

P_2 : The wife knows that the probability of the doctor being an expert is 50%.

P_3 : The wife believes that the probability that the good news actually came from the expert is larger if the husband recovers than if he does not, i.e., $\pi_T(\cdot|s_1) \neq \pi_T(\cdot|s_2)$.

Such proxies are called *evidence with stochastic reliability.* A somewhat similar idea has been used in a different context by Thaler (2024). ◁

Note that $(P_0) - (P_2)$ are all exogenous assumptions, in the sense that they cannot be corroborated with traditional choice data. In particular, (P_0) is exogenous for the same reason the assumption of state-independent utilities across S is also exogenous, i.e., we can only justify it on the basis of accepting that, upon fixing any $s \in S$, the agent has no intrinsic preference over realizations of T, nor does she have any unobservable side payments that depend on T. Condition (P_1) is also exogenous, as it makes direct reference to the actual belief μ, which is of course unidentifiable with traditional choice data. Finally, (P_2) is exogenous because it is an assumption regarding the unconditional belief about T, which is in turn a function of μ, as S and T are correlated. Such correlation also explains why we cannot directly elicit π_T, instead of exogenously assuming it to be equal to π_T^{obj}.

On the other hand, once we have assumed that (P_0) holds, (P_3) can be tested with traditional choice data. This is because under (P_0), the conditional beliefs $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ can be directly identified (Section 4), and once these are known it can be simply checked whether they are linearly independent. The formal distinction between exogenous and testable conditions is further discussed in Section 6, where I provide decision-theoretic foundations.

Then, my main result shows that we can leverage conditional beliefs about the proxy to identify the actual belief about S.

Theorem 1 (Main identification result)**.** *Assume that T satisfies $(P_0)-(P_2)$. Then, the actual belief $\mu \in \Delta(S)$ is uniquely identified if and only if (P_3) is satisfied.*

The crucial step towards identification is to pin down the joint belief π. To do this, first, we identify the conditional beliefs $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$, leveraging the fact that the agent has no stakes in T conditional on S. Then, we mix these conditional beliefs with π_S, and we calibrate the marginal π_T against the objective marginal π_T^{obj}, similarly in spirit to the idea of Anscombe and Aumann (1963) who calibrate subjective probability distributions against objective ones.[7] This calibration exercise allows us to identify π_S, and subsequently to obtain the joint belief π. Finally, by conditioning with respect to the uninformative event $E \subseteq T$ we obtain μ. Let me provide an illustration.

Example 4 (continued). Suppose that the wife reports probability $\pi_T(t_1|s_1) = 0.80$ of the husband receiving the drug conditional on recovering, and probability $\pi_T(t_1|s_2) = 0.40$ of the husband receiving the drug conditional on remaining paralyzed. There is a unique convex combination of the two aforementioned conditional probabilities inducing the objectively known $\pi_T(t_1) = 0.50$, i.e., we have $0.25\pi_T(\cdot|s_1) + 0.75\pi_T(\cdot|s_2) = \pi_T$. So, it is necessarily the case that $\pi_S(s_1) = 0.25$, meaning that the joint belief π is given by the following table:

	recovers (s_1)	paralyzed (s_2)
drug (t_1)	0.20	0.30
placebo (t_2)	0.05	0.45

Hence, her actual belief attaches probability $\mu(s_1) = \pi_S(s_1|t_2) = 0.10$ to her husband recovering. Note that the identification would not have been possible if $\pi_T(\cdot|s_1) = \pi_T(\cdot|s_2)$ (i.e., if S and T were independent), as in this case we would have obtained $\lambda\pi_T(\cdot|s_1) + (1-\lambda)\pi_T(\cdot|s_2) = \pi_T$ for every $\lambda \in (0, 1)$. ◁

Importantly, for my identification strategy to work, it must be the case that $|\operatorname{supp}(\pi_T)| \geq |S|$. Otherwise, (P_3) will be directly violated, since $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ will not be linearly independent, and a fortiori we will not be able to identify the joint belief π. The good news is that the amount of data that we need is still quite small, e.g., whenever the state space is binary, we only need to elicit two probabilities.

Now that my identification method is clear, let me go back to the relationship with the one of Lu (2019). Technically, the two approaches are similar in that both introduce an additional dimension T, and subsequently take as input the conditional distributions $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ and the marginal π_T, in order to obtain the joint distribution π. Finally both papers eventually condition with an uninformative event, which in Lu's case is always the entire set T. At the same time, the two approaches crucially differ in terms of where these inputs come from. In Lu (2019), the marginal distribution is approximated by the frequency of observed choices, while the conditional distributions are either exogenously given (when a single signal is used) or inferred from the data (when two ordered signals are used). On the other hand, in my paper things are upside down, i.e., the conditional distributions are directly elicited and the marginal is exogenously given. These differences are natural, given that Lu (2019) focuses on a specific set of applications, where a large number of choices is readily available, conditional on independent signal realizations from T. Such applications include hiring decisions, loan approvals, medical advice. However, this requirement rules out many cases where we want to elicit the agent's belief about a one-shot realization of S, e.g., if

[7] I would like to thank an anonymous referee for pointing out this connection.

we are interested in the wife's belief in our running example, or in some partisan's belief about the winner of the upcoming presidential election.

On a high level, my approach bears a striking conceptual similarity to the use of instrumental variables in regression analysis. In particular, instead of automatically imposing an exogenous assumption (viz., orthogonality) on their variable of interest (viz., explanatory variable), econometricians assume it on another carefully-chosen variable (viz., instrumental variable). Similarly, instead of exogenously assuming without much thought that the agent has no stakes about the original state space (viz., S), I exogenously assume $(P_0) - (P_2)$ about a carefully-chosen proxy (viz., T). That is, in both cases, it is not that we stop imposing exogenous assumptions all together, but rather we only do so in specifically-chosen domains where the exogenous assumptions can be justified on the basis of common sense and/or well-established theoretical insights.

Such flexibility is what has made instrumental variables so successful. Similarly, I argue that my belief identification approach via proxies is appealing because there exists abundance of potentially suitable proxies, one of which will eventually satisfy common sense and/or will be consistent with well-established insights. That is, although $(P_0) - (P_2)$ might seem demanding ex ante, this is not really a problem, as we only need to find one proxy for which we will be comfortable with imposing these assumptions. For example, in case we find it difficult to justify (P_1) in Example 4 because of the placebo effect, we can switch to Example 5 where we are more comfortable with assuming that the wife does not have any information on the group of men that carries the gene. And at the end of the day, even if none of these readily available proxies satisfies us, we can always construct one specifically for the purpose of our identification exercise (Section 7).

4. Belief elicitation mechanisms

The common practice for identifying actual beliefs is by using belief elicitation mechanisms. These are direct mechanisms that incentivize the agent to reveal her actual belief (about the original state space S), by rewarding accurate reports and punishing inaccurate ones. A mechanism is formally written as a function

$$\sigma : \Delta(S) \to \mathcal{F}_S,$$

that takes as input the agent's reported belief $r \in \Delta(S)$, and returns as output a consequence $\sigma^r(s) \in Q$ upon state $s \in S$ being realized.

An elicitation mechanism is strictly incentive-compatible if truthful revelation of the actual belief is the agent's only optimal report, i.e., formally, for any actual belief $\mu \in \Delta(S)$, act $\sigma^\mu \in \mathcal{F}_S$ which is associated with reporting truthfully is strictly preferred to act $\sigma^r \in \mathcal{F}_S$ which is associated with any other report $r \neq \mu$. Existing mechanisms within the literature differ in how the incentives are framed.

The most popular belief elicitation mechanism is the *quadratic scoring rule* (De Finetti, 1937; Brier, 1950; Good, 1952). This is a direct mechanism that rewards the agent with a flat payment minus an inaccuracy penalty which is proportional to the squared (Euclidean) distance between the submitted report and the realized state. Formally, if the agent reports r and state s is realized, she will be paid the monetary amount

$$\sigma^r(s) = \alpha - \beta \|r - \mathbf{1}_s\|^2, \tag{6}$$

where $\beta > 0$ and $\mathbf{1}_s$ is the Dirac measure that puts probability 1 to state s.

The widely-accepted claim is that —whenever the agent is risk-neutral— quadratic scoring rules are strictly incentive-compatible.[8] Unfortunately, there is a subtle misconception which goes back to the belief identification problem, viz., without the assumption of state-independent utilities, the quadratic scoring rule is not incentive-compatible, even if the agent is risk-neutral.

Example 7. (Quadratic Scoring Rule in the wife's problem). Recall the wife's problem from Example 1, and suppose that the wife is asked to report a probability $r(s_1) \in [0,1]$ of her husband recovering. In return, a quadratic scoring rule will determine her payment according to formula (6). Now, suppose that the pair (u, μ) is a SEU representation which involves her actual belief μ. Then, the wife will submit a report that maximizes the following function:

$$\mathbb{E}_\mu\big(u(\sigma^r)\big) = \mu(s_1)\gamma_1\Big(\alpha - 2\beta\big(1 - r(s_1)\big)^2\Big) + \mu(s_2)\gamma_2\Big(\alpha - 2\beta\big(1 - r(s_2)\big)^2\Big).$$

The first order condition yields the following optimality condition

$$\frac{\mu(s_1)}{\mu(s_2)} = \frac{r(s_1)}{r(s_2)} \cdot \frac{\gamma_2}{\gamma_1}, \tag{7}$$

which in turns means that the wife will report her actual belief in response to the incentives provided by the quadratic scoring rule if and only if $\gamma_1 = \gamma_2$, or equivalently if and only if (u, μ) is a SI-SEU representation. ◁

The previous example suggests that the entire applied literature that uses quadratic scoring rules to elicit beliefs relies on the implicit assumption that the subject (viz., the agent in our case) does not have any stakes in the underlying state space. This is because submitting a report corresponds to choosing an act from the menu

$$\{\sigma^r | r \in \Delta(S)\} \subseteq \mathcal{F}_S. \tag{8}$$

And, as we have already discussed extensively, such choice data cannot help us identify the agent's actual belief unless we assume state-independent utilities.

The need to assume state-independent utilities is not exclusive to cases where we use the quadratic scoring rule. This is because almost all elicitation mechanisms can be rewritten as menus of acts, like in (8). This is for instance the case for other scoring rules with deterministic rewards, binarized scoring rules, Karni mechanisms, matching probabilities, just to mention the most common ones.[9] In the end, all these mechanisms yield traditional choice data, from which we cannot identify the agent's actual belief without running into the identification problem.[10]

Remark 2. Within the mechanism design literature, utility from money is typically linearly separable from the utility component that depends on the opponents' type. As a result, we can use standard belief elicitation tools to elicit one's beliefs about everyone else's type, despite being in a setting with interdependent types (Johnson et al., 1990; Jehiel et al., 2012). This is possible for the same

[8] In the scoring rule literature, an incentive-compatible (resp., strictly incentive-compatible) mechanism is called proper (resp., strictly proper).

[9] Examples of scoring rules with deterministic rewards include the logarithmic scoring rule and the spherical scoring rule (Good, 1971). There is a general characterization of this entire class by means of strictly convex indirect expected payoff functions (Savage, 1971). Binarized scoring rules pay in probability units to win a fixed prize in order to relax the risk-neutrality assumption (Hossain and Okui, 2013). Binarized scoring rules belong to the class of stochastic mechanisms, which also includes the Karni mechanisms (Karni, 2009; Tsakas, 2019) and the method of matching probabilities (Ducharme and Donnell, 1973; Baillon et al., 2018). For overviews of this literature, see Schlag and van der Weele (2013) and Schotter and Trevino (2014).

[10] The only exception is a handful of mechanisms which utilize non-traditional choice data (Karni, 1999; Jaffray and Karni, 1999). Nevertheless, these mechanisms are quite cumbersome and data-demanding.

reason we can uniquely identify beliefs if we assume that utilities are quasi state-independent, as pointed out in Footnote 6. And vice versa, the same way we cannot pin down beliefs without an exogenous assumption —like quasi state-independent utilities— we will not be able to elicit beliefs about the opponents' types in mechanism design without the separability property which is implied by quasi-linearity.[11] ◁

While the traditional mechanisms cannot help us with directly eliciting beliefs about the original state space S, they can still be used for eliciting the conditional beliefs about a proxy T given the original state space S. In practice, the agent will be asked to submit K reports $r_1, \ldots, r_K$, with $r_k \in \Delta(T)$ being her reported conditional belief about T given s_k. She will only be paid for her report r_k based on an incentive-compatible mechanism

$$\sigma_k : \Delta(T) \to \mathcal{F}_T,$$

if state s_k is eventually realized, e.g., if σ_k is a quadratic scoring rule, the agent will receive

$$\sigma_k^{r_k}(t) = \alpha - \beta \|r_k - \mathbf{1}_t\|^2.$$

Note that in the end, the agent will only be paid for one of these mechanisms, as only one s_k will be eventually realized, meaning that this is essentially an implementation of the strategy method.[12]

5. Actual utility function

In this section, I will show how my main identification result allows me to obtain a well-founded definition of an *actual (state-dependent) utility function*. This is a very useful object, as it provides meaning to statements like "a dollar is twice as valuable at s_1 compared to s_2" or "the utility of state s is equal $w(s)$", which are in turn commonly used in many applied settings, e.g., in the literature on motivated beliefs. Furthermore, actual utilities will be relevant for a recent literature that focuses on estimating tastes using directly elicited beliefs.

Note that these results will not be exclusive to my setting, and will also apply to other SEU models that uniquely identify actual beliefs. However, surprisingly, the point is largely ignored so far in the literature. I speculate that this is because most of the existing literature does not make the explicit distinction between "actual beliefs" and "beliefs that are uniquely identified within a SEU model". This distinction is necessary in order to identify "actual utilities" and is further discussed in Section 6.

Back in Example 1, recall that every SEU representation $(\tilde{u}, \tilde{\mu})$ of the wife's preferences satisfies

$$\frac{\tilde{\mu}(s_2)}{\tilde{\mu}(s_1)} = \frac{\tilde{\gamma}_1}{9\tilde{\gamma}_2},$$

where $\tilde{u}_1(q) = \tilde{\gamma}_1 q$ and $\tilde{u}_2(q) = \tilde{\gamma}_2 q$. Therefore, there is a duality between identification of beliefs and identification of relative marginal utilities. This exact duality is what the literature traditionally leverages to go from state-independent utilities to a unique belief, viz., by assuming $\gamma_1 = \gamma_2$ one can uniquely identify $\bar{\mu}(s_1) = 90\%$. In this paper, on the other hand, we will use this duality the other way around, viz., once we have identified the actual belief (using Theorem 1), we can uniquely identify a set of parameters (γ_1, γ_2) that will constitute the "actual utility function". For instance,

[11] Once again, I would like to thank the same referee that made the comment about quasi state-independent utilities.

[12] The fact that there is only one payment guarantees that there are no hedging opportunities (Blanco et al., 2010) which is a common concern in cases where the agent faces multiple decisions over acts.

in our running example, the actual belief is $\mu(s_1) = 0.10$ (Example 4, continued), and therefore the actual utility function can be written as

$$u_1(q) = \underbrace{81\beta}_{\gamma_1} q \text{ and } u_2(q) = \underbrace{\beta}_{\gamma_2} q, \tag{9}$$

where $\beta > 0$. The following theorem generalizes this result beyond my simple example, even in cases where there is no SI-SEU representation.

Theorem 2. *Let $(\tilde{u}, \tilde{\mu})$ be an arbitrary SEU representation. Moreover, assume that the agent's actual belief $\mu \in \Delta(S)$ has been identified (via Theorem 1). Then, the class of utility functions $u : S \times Q \to \mathbb{R}$ for which (u, μ) is a SEU representation is characterized by*

$$u_s = \alpha_s + \beta \frac{\tilde{\mu}(s)}{\mu(s)} \tilde{u}_s, \tag{10}$$

where $\alpha_s \in \mathbb{R}$ and $\beta > 0$, for all $s \in S$.

It should be stressed that throughout the paper the primitives are the preferences and the actual beliefs, while the actual utilities are still derived within the context of a SEU model. Using the actual utility function is only needed in order for us (i.e., the analyst) to correctly associate the agent's choices among acts with her actual belief.

Not surprisingly, as this is a Bayesian framework, the arrival of new information affects the actual beliefs, but not the actual tastes. So, the set of actual utility functions remains invariant when the agent updates her beliefs upon observing new information about S.

Remark 3. Interestingly, if the preferences admit a SI-SEU representation $(\bar{u}, \bar{\mu})$, Theorem 2 implies that the SEU function becomes

$$\mathbb{E}_\mu\big(u(f)\big) = \sum_{s \in S} \mu(s) w(s) \bar{u}(f_s), \tag{11}$$

where $w(s) = \bar{\mu}(s)/\mu(s)$ is the utility from state s. So, although state-independent preferences do not identify the actual belief, they still allow us to separately identify (state-independent) utility from a consequence at each state and utility from the state itself. ◁

6. Decision-theoretic foundations

In this section, I provide axiomatic foundations for the conditions that appear in Definition 1, in order to clarify which assumptions are exogenous and which are testable with traditional choice data.

Aligned with Anscombe and Aumann (1963), let $Q = \Delta(X)$ be the set of lotteries over a finite set of prizes X.[13] Take the extended state space $S \times T$, and denote the set of all acts by $\mathcal{F} := Q^{S \times T}$, with $f_{s,t} := f(s,t)$ being the lottery attached to state (s,t) by act f. The set of S-measurable acts is denoted by $\mathcal{F}_S$.[14] Compound acts are defined in the usual way, i.e., $\lambda f + (1-\lambda) g$ induces the outcome $\lambda f_{s,t} + (1-\lambda) g_{s,t}$ at state $(s,t) \in S \times T$. For any $f, g \in \mathcal{F}$ and any $A \subseteq S \times T$, define the act $f_A g$ by

$$(f_A g)(s,t) = \begin{cases} f(s,t) & \text{if } (s,t) \in A, \\ g(s,t) & \text{if } (s,t) \notin A, \end{cases}$$

[13] The analysis can be easily extended to other decision-theoretic frameworks, e.g., Savage (1954) or Wakker (1989).

[14] As usual, an act $f \in \mathcal{F}$ is S-measurable (resp., T-measurable) whenever $f(s,t) = f(s,t')$ for all $s \in S$ and $t, t' \in T$ (resp., $f(s,t) = f(s',t)$ for all $s, s' \in S$ and $t \in T$).

i.e., $f_A g$ coincides with f in A and with g everywhere else.

The agent has (weak) preferences $\succeq$ over $\mathcal{F}$. As usual, $\succ$ and $\sim$ denote the asymmetric part (viz., strict preference) and the symmetric part (viz., indifference) respectively. Conditional preferences $\succeq_A$ are defined in the usual way: $f \succeq_A g$ if and only if $f_A h \succeq g_A h$ for all $h \in \mathcal{F}$. Throughout the paper, for notational simplicity, we write $\succeq_s := \succeq_{\{s\}\times T}$ and $\succeq_{s,t} := \succeq_{\{s\}\times\{t\}}$.

Note that the preference relation $\trianglerighteq$ that we considered in Section 2 is not necessarily the same as $\succeq$ restricted in $\mathcal{F}_S$, but rather as the conditional preference relation $\succeq_{S\times E}$ restricted in $\mathcal{F}_S$. To see that this the case, recall Example 4, where $\succeq \neq \trianglerighteq$ and $\succeq_{t_2} = \trianglerighteq$, i.e., the wife's original preferences over $\mathcal{F}_S$ are the same as the ones she would have had conditional on the husband having taken the placebo, but not unconditionaly.

The latter also sheds some light on the relationship between this work and the distinction between small and large worlds of Savage (1954). On the one hand, similarly to Savage, introducing a proxy allows us to consider more contingencies, in that we also look at preferences over $(S \times T)$-measurable acts, rather just S-measurable acts. That is, in the words of Savage (1954), we go from the smaller world S to the larger world $S \times T$. At the same time, in the small world we implicitly condition with respect to the event E. In this sense, unlike in Savage, when we go from the small to the large world, we are not only enriching the language (by considering more relevant aspects), but we are also adding more states that were previously ruled out from our consideration.

An event $A \subseteq S \times T$ is called null if for any two acts $f, g \in \mathcal{F}$ we have $f \sim_A g$. For the time being, to simplify presentation, we assume that there are no null states:[15]

(A_0) Full support — No state $(s,t) \in S \times T$ is null.

Consider the following standard axioms:

(A_1) Completeness — For all $f, g \in \mathcal{F}$: $f \succeq g$ or $g \succeq f$.

(A_2) Transitivity — For all $f, g, h \in \mathcal{F}$: if $f \succeq g$ and $g \succeq h$, then $f \succeq h$.

(A_3) Continuity — For all $f, g \in \mathcal{F}$: $\{g \in \mathcal{F} : f \succeq g\}$ and $\{g \in \mathcal{F} : g \succeq f\}$ are closed in $\mathcal{F}$.

(A_4) Independence — For all $f, g, h \in \mathcal{F}$, and for all $\lambda \in (0,1)$: $f \succeq g$ if and only if $\lambda f + (1-\lambda)h \succeq \lambda f + (1-\lambda)h$.

These axioms form the basic premise for connecting beliefs to choices through some SEU representation (Karni et al., 1983). Formally, $(A_1)-(A_4)$ are satisfied if and only if a linear (state-dependent) utility function $u : S \times T \times Q \to \mathbb{R}$ together with actual joint belief $\pi \in \Delta(S \times T)$, represent the preferences $\succeq$ via the SEU

$$\mathbb{E}_\pi\big(u(f)\big) = \sum_{s\in S}\sum_{t\in T} \pi(s,t) u_{s,t}(f_{s,t}). \tag{12}$$

That is, for all $f, g \in \mathcal{F}$,

$$f \succeq g \;\Leftrightarrow\; \mathbb{E}_\pi\big(u(f)\big) \geq \mathbb{E}_\pi\big(u(g)\big). \tag{13}$$

Linearity of u implies that there is some (state-dependent) vNM utility function $v : S \times T \times X \to \mathbb{R}$, such that for every $(s,t) \in S \times T$ and every $q \in \Delta(X)$, the utility over consequences in (12) can be

[15] Without exogenously assuming that there are no null states, the definition of a proxy needs to be slightly adjusted. First of all, observe that the support of π can be uniquely identified from $\succeq$. Then, once we have pinned down the set of non-null states, conditions (P_0), (P_1) and (P_3) will be respectively replaced by the following assumptions: (1) the agent has no stakes in T conditional on any non-null $s \in S$, (2) the uninformative event $E \subseteq T$ is non-null, and (3) the vectors in $\{\pi_T(\cdot|s) | s \text{ is non-null}\}$ are linearly independent.

rewritten as

$$u_{s,t}(q) = \sum_{x \in X} q(x) v_{s,t}(x).$$

For the rest of the paper we will work with u, keeping in mind that it is linear.

If we add (A_0) on top of $(A_1)-(A_4)$, the belief π becomes full-support. Of course, the identification problem still holds, i.e., for any $\tilde{\pi} \in \Delta(S \times T)$ there exists some linear $\tilde{u}$ such that $\mathbb{E}_\pi(u(f)) = \mathbb{E}_{\tilde{\pi}}(\tilde{u}(f))$ for all $f \in \mathcal{F}$, meaning that $(\tilde{u}, \tilde{\pi})$ also represents $\succeq$. I henceforth treat $(A_0) - (A_4)$ as prerequisite assumptions, not because they cannot be tested, but rather because the belief identification problem is formally defined only once they hold.

Then, assuming that $(A_0) - (A_4)$ are satisfied, I will proceed to restrict the class of SEU representations that are consistent with T being a proxy for S, given that π_T^{obj} has been already labelled as *the objective marginal belief.* The latter means that there are commonly known objective facts from which π_T^{obj} is obtained.

Definition 2. A full-support SEU representation $(\bar{u}, \bar{\pi})$ is called *Conditionally State-Independent (abbrev., CSI-SEU)* whenever $\bar{u}$ is S-measurable, i.e., whenever $\bar{u}_{s,t}(q) = \bar{u}_{s,t'}(q)$ for all $s \in S$, all $t, t' \in T$, and all $q \in Q$. Furthermore, a CSI-SEU representation $(\bar{u}, \bar{\pi})$ is called *Proxy-Consistent (abbrev., PC-SEU)*, whenever (a) $\bar{\pi}_T = \pi_T^{\text{obj}}$, and (b) $\bar{\pi}_T(\cdot|s_1), \dots, \bar{\pi}_T(\cdot|s_K)$ are linearly independent.

Being able to represent $\succeq$ with a PC-SEU $(\bar{u}, \bar{\pi})$ is the minimal necessary condition for T to be a proxy, viz., S-measurability of $\bar{u}$ is necessary for (P_0); agreement of $\bar{\pi}_T$ with π_T^{obj} is necessary for (P_2); and linear independence of $\bar{\pi}_T(\cdot|s_1), \dots, \bar{\pi}_T(\cdot|s_K)$ is necessary for (P_3). However, existence of a PC-SEU representation is not sufficient by any means, because $(P_0) - (P_3)$ involve exogenous assumptions. For instance, even if we establish that $(\bar{u}, \bar{\pi})$ is a PC-SEU representation, it is never possible to conclude that $\bar{\pi}$ is the actual joint belief π, for the same reasons we could not conclude $\bar{\mu} = \mu$. In this sense, we can only reject the hypothesis that T is a proxy by showing that $\succeq$ does not have a PC-SEU representation, but we cannot verify it even if a PC-SEU representation exists. I further discuss this issue at the end of this section. For the time being, I will axiomatize the class of PC-SEU representations, beginning with the following axiom:

(A_5) Local state-monotonicity — For all $s \in S$, for all $t, t' \in T$, and for all $p, q \in Q : p \succeq_{s,t} q$ if and only if $p \succeq_{s,t'} q$.

This is simply a weakening of the standard state-monotonicity axiom (Anscombe and Aumann, 1963), in that it postulates state-independent preferences only across states in $\{s\} \times T$, rather than across all states in $S \times T$. It is not difficult to show that, together with $(A_0) - (A_4)$, it axiomatizes CSI-SEU. But most importantly, analogously to Anscombe and Aumann (1963), it guarantees that the conditional belief $\bar{\pi}_T(\cdot|s)$ is uniquely identified for every $s \in S$, i.e., all CSI-SEU representations yield the same $\bar{\pi}_T(\cdot|s_1), \dots, \bar{\pi}_T(\cdot|s_K)$ (Lemma A1).

In order to formally state my next two axioms, I first need to introduce some additional notation. Let $\mathcal{E} = \{f^{\text{high}}, f^{\text{low}}\}$ be a binary menu of strict-dominance-ordered S-measurable acts, i.e., for all states (s, t), it is the case that $f_{s,t}^{\text{high}} =: f_s^{\text{high}}$ and $f_{s,t}^{\text{low}} =: f_s^{\text{low}}$, and moreover

$$f^{\text{high}} \succ_{s,t} f^{\text{low}}.$$

For instance, if $\succeq$ satisfies (A_5), one such menu is formed if we take f^{high} and f^{low} to be a $\succeq$-maximal and a $\succeq$-minimal act respectively. If such menu exists, consider the lattice (with respect to the dominance relation),

$$\mathcal{F}_{\mathcal{E}} := \{f \in \mathcal{F} : f^{\text{high}} \succeq_{s,t} f \succeq_{s,t} f^{\text{low}} \text{ for all } (s,t) \in S \times T\}. \tag{14}$$

Unlike $\mathcal{E}$ itself, $\mathcal{F}_{\mathcal{E}}$ does not necessarily contain only S-measurable acts. For instance, in our earlier example (where f^{high} and f^{low} are $\succeq$-maximal and $\succeq$-minimal acts respectively), $\mathcal{F}_{\mathcal{E}}$ is the entire set $\mathcal{F}$. Since at least one of the two relations in (14) is strict, by continuity, for every $f \in \mathcal{F}_{\mathcal{E}}$ and every state (s,t) there exists a unique $\lambda_{s,t}^{\mathcal{E},f} \in [0,1]$ such that

$$f \sim_{s,t} (1-\lambda_{s,t}^{\mathcal{E},f}) f^{\text{low}} + \lambda_{s,t}^{\mathcal{E},f} f^{\text{high}}. \tag{15}$$

Thus, each $f \in \mathcal{F}_{\mathcal{E}}$ is associated with a unique vector $\lambda^{\mathcal{E},f} \in [0,1]^{S\times T}$. Then, define

$$\mathcal{T}_{\mathcal{E}} = \{f \in \mathcal{F}_{\mathcal{E}} : \lambda^{\mathcal{E},f} \text{ is } T\text{-measurable}\}. \tag{16}$$

In general, T-measurability of $\lambda^{\mathcal{E},f}$ neither implies nor is it implied by T-measurability of f.

Finally, for every $f \in \mathcal{F}$, define the S-measurable act

$$f_s^{\text{obj}} := \sum_{t\in T} \pi_T^{\text{obj}}(t) f_{s,t}. \tag{17}$$

The idea is that f^{obj} averages across the lotteries that f assigns to the different states in $\{s\}\times T$ with respect to the exogenously given objective marginal π_T^{obj}, and subsequently assigns to each state (s,t) the average lottery f_s^{obj}.

Then, I am ready to introduce the two new axioms:

(A_6) Objective mixture indifference — There is some binary strict-dominance-ordered $\mathcal{E} \subseteq \mathcal{F}_S$ such that for all $f \in \mathcal{T}_{\mathcal{E}} : f^{\text{obj}} \sim f$.

(A_7) Unique extension — For any binary strict-dominance-ordered $\mathcal{E} \subseteq \mathcal{F}_S$, and for any $\succeq'$ satisfying $(A_0)-(A_5)$: if $\succeq \,=\, \succeq'$ in $\mathcal{T}_{\mathcal{E}}$ then $\succeq \,=\, \succeq'$ in $\mathcal{F}_{\mathcal{E}}$.

Axiom (A_6) will guarantee that, there is a CSI-SEU representation $(\bar{u},\bar{\pi})$ such that $\bar{\pi}_T = \pi_T^{\text{obj}}$ (Lemma A2), i.e., it essentially allows us to calibrate against the objective marginal π_T^{obj}, similarly in spirit to the calibration exercise of Anscombe and Aumann (1963). This axiom is of somewhat non-standard nature as it is defined with respect to the exogenously given π_T^{obj}. It essentially postulates that if we replace every conditional belief $\bar{\pi}_T(\cdot|s)$ with the objective belief π_T^{obj}, the implied preferences over $\mathcal{T}_{\mathcal{E}}$ will not change.

Axiom (A_7) will guarantee that the uniquely identified beliefs $\bar{\pi}_T(\cdot|s_1),\dots,\bar{\pi}_T(\cdot|s_K)$ that have been previously obtained as part of every CSI-SEU representation are linearly independent (Lemma A3).

Theorem 3. *There exists a PC-SEU representation $(\bar{u},\bar{\pi})$ if and only if $\succeq$ satisfies $(A_0)-(A_7)$. Furthermore, $\bar{\pi}$ is uniquely identified from $\succeq$.*

The first important implication of the previous result is that it allows us to formally state the exogenous assumptions in the definition of a proxy. These assumptions are: (1) the objective marginal belief π_T^{obj} agrees with the actual marginal belief π_T, (2) the conditional beliefs $\bar{\pi}_T(\cdot|s_1),\dots,\bar{\pi}_T(\cdot|s_K)$ agree with the actual conditional beliefs $\pi_T(\cdot|s_1),\dots,\pi_T(\cdot|s_K)$, and (3) the conditional belief $\bar{\pi}_S(\cdot|E)$ agrees with the actual belief μ. Not surprisingly, all three of them have to do with the relationship between $\bar{\pi}$ and the actual beliefs.

The second important implication of Theorem 3 goes back to the long-standing debate on the definition of subjective probability (e.g., Grant and van Zandt, 2008; Karni, 2014, and references therein). Although I personally subscribe to the more modern approach (according to which actual beliefs exist), this paper can be easily rewritten along the lines of the classical view (according to which the only observable primitive is the preference relation, and therefore beliefs should be simply defined as the probability measure that we uniquely identify within a class of SEU representations). Indeed, using Theorem 3, I could have simply labelled the uniquely identified $\bar{\pi}_S(\cdot|E)$ as the agent's belief, without going into the whole discussion on whether this is her actual beliefs, or even on whether an actual belief exists in the first place.

7. Belief identification in practice

7.1. Availability of proxies

Suitable proxies can be classified into two broad categories, depending on whether we rely on existing observational data or on experimental data that we specifically collect for the purpose of our elicitation exercise.

Starting with the former, sometimes a suitable proxy appears spontaneously, like for instance in Example 4, where a clinical trial incidentally happens to take place. Other times a suitable proxy is picked from publicly available statistics, like for instance in Example 5, where the percentage of people who carry the specific gene is known. Such proxies will often fit our definition well, but it is unclear whether they will always be readily available.

In this last case we turn to the second category, where we design an experiment —prior to the elicitation task— with the specific aim of constructing a suitable proxy. Fortunately for us, it is essentially always feasible to construct such a proxy. To do this, we first generate an informative signal, then we distort it, and finally we elicit the agent's conditional probabilities of the signal having been distorted given each realization of the original state space, as I have already briefly described in Example 6. Let me elaborate a bit further in order to showcase the easiness of constructing such proxies.

Begin with collecting (conditionally) independent predictions about the husband's health outcome from a group of experts. Examples of such experts include doctors, medical students, AI tools, etc. The wife knows which experts are in the group, but does not know the prediction of each individual expert. From her point of view, the informative signal is given by her perceived reliability of a randomly drawn expert, i.e., by the subjective probability that she attaches to a randomly picked expert correctly predicting her husband's recovery, as well as the subjective probability that she assigns to a randomly picked expert correctly predicting that her husband remains paralyzed (Figure 2). Importantly, we remain agnostic on what the wife thinks about what these error probabilities

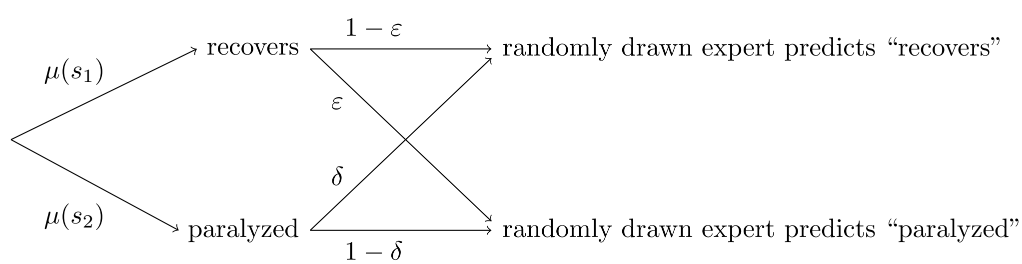

Figure 2: The wife's perceived reliability of a randomly drawn expert are described by the two error probabilities, i.e., ε and δ, which correspond her subjective estimate of a random expert's type 1 and type 2 error.

could look like. But at the end of the day this is not a problem for our purposes, as we only care about the wife believing that the signal contains some information, i.e., our only requirement is

$1-\varepsilon \neq \delta$. This means, for instance, that we do not require ε and δ to coincide with an expert's actual reliability.

Now, we distort the signal by mixing the prediction of a randomly drawn expert with noise. Here is one of the many potential examples on how we could implement such distortion in practice. Suppose that inside an urn we place a green ball for each expert predicting "recovers" and a red ball for each expert predicting "paralyzed". All these balls will be henceforth called informative, as each of them corresponds to an actual prediction. Then, we add into the same urn some more balls, half of them being green and half being red. We call these balls uninformative, as they do not correspond to anyone's prediction. Then, we define the set

$$T = \{\text{informative ball } (t_1),\ \text{uninformative ball } (t_2)\}.$$

The wife's joint belief over states and signal realizations, given an informative ball (resp., given an uninformative ball), is the distribution on the left hand-side table (resp., right hand-side table):

	recovers (s_1)	paralyzed (s_2)
green ball	$(1-\varepsilon)\mu(s_1)$	$\delta\mu(s_2)$
red ball	$\varepsilon\mu(s_1)$	$(1-\delta)\mu(s_2)$

informative ball (t_1)

	recovers (s_1)	paralyzed (s_2)
green ball	$0.5\mu(s_1)$	$0.5\mu(s_2)$
red ball	$0.5\mu(s_1)$	$0.5\mu(s_2)$

uninformative ball (t_2)

This follows directly from Figure 2 above.

Suppose that a ball is randomly drawn from the urn. For illustration purposes, let this ball be green. The wife is told that the ball is green. She is also told the proportion of informative balls among the green balls, which is henceforth denoted by $\pi_T^g(t_1)$. Then, her joint belief over the product space $S \times T$ will be given by the following table:

	recovers (s_1)	paralyzed (s_2)
informative ball (t_1)	$\pi_T^g(t_1)\frac{(1-\varepsilon)\mu(s_1)}{(1-\varepsilon)\mu(s_1)+\delta\mu(s_2)}$	$\pi_T^g(t_1)\frac{\delta\mu(s_2)}{(1-\varepsilon)\mu(s_1)+\delta\mu(s_2)}$
uninformative ball (t_2)	$\pi_T^g(t_2)\mu(s_1)$	$\pi_T^g(t_2)\mu(s_2)$

Note that all the assumptions of Definition 1 are consistent with this joint belief, as long as $1-\varepsilon \neq \delta$, meaning that T can be used as a suitable proxy. In particular, as required by (P_1), the event $\{t_2\}$ is uninformative. Moreover, in line with (P_2), the marginal distribution $\pi_T^g \in \Delta(T)$ is commonly known. Finally, as postulated by (P_3), the proxy T is correlated with S. Thus, given that it is also reasonable to also assume (P_0) (see Example 6), we are comfortable with using T as a proxy.

Concluding, it is exactly the flexibility in picking any information sources for generating the signal, combined with the fact that we do not impose any assumption on what the wife believes about the source's reliability, what makes this approach essentially always feasible.

7.2. Applications revisited

As pointed out in the introduction, there are several applications where state-dependent preferences can potentially lead to measurement error when eliciting beliefs. Let me illustrate here how my proxy method could be useful in mitigating this problem in a couple of those application.

Suppose that we want to elicit the beliefs of partisan voters about the outcome of the upcoming presidential election. It is well known that subjects systematically differ in their stated beliefs depending on which candidate they support. The question is whether such differences should be fully attributed to overconfidence —as the literature on politically motivated beliefs suggests— or whether part of these differences should be attributed to measurement error because of state-dependent preferences. If such measurement error is positive, we can conclude that the standard methodology overestimates the effect of motivation. But before explaining how my method can help us solve the problem, let me explain why the identification problem is likely to arise here. The preference that partisan voters exhibit for their own candidate winning can be associated with expectation of some personal gain in case this happens, e.g., otherwise they may choose to relocate abroad in which case their purchasing power will drop, or they may suffer psychologically and consumption is a way to compensate. In either case, it is likely that their utility from money is state-dependent.

Having said this, within this application, we can always use a variant of the experimental design we presented in Section 7.1 above, i.e., we can always present the voter with a forecast or a poll, and then elicit her conditional beliefs about this forecast/poll coming from an actual source conditional on her preferred candidate winning, as well as conditional on the opponent winning.

Now, let me turn to a second application from the ones I presented in the introduction. Suppose that we want to use the forecast of a financial analyst as input for our decision on whether to buy a certain stock or not. However, although we have calibrated this analyst's utility for money, we cannot observe her existing portfolio and therefore we do not know whether she has already invested in this stock. Thus, the analyst's state-dependent preferences enter the picture in the form of unobservable side payments, like in Example 3.

Within this application, we can again use an experimental design along the lines of what I presented in Section 7.1. In particular, suppose that I present the analyst with the forecast of another expert, and subsequently elicit her conditional beliefs of this other forecast coming from an actual expert conditional on the stock's price going up, as well as conditional on the stock's price going down.

8. Conclusion

In this paper, I proposed a novel approach for identifying subjective beliefs without exogenously imposing the awkward assumption of state-independent utilities. The idea is to enlarge the state space by introducing a second dimension, which I call a proxy. The key property of a proxy is that the agent has no stakes in its realization conditional on the original state space. It is exactly this property that allows us to uniquely identify the agent's conditional beliefs about the proxy given each realization of the original state space, using a variant of the strategy method. This method is less data-demanding and more flexible compared to other methods in the literature. Thus, it is not just theoretically sound, but also tractable. Of course, there are various important details which pertain to the experimental implementation of the methodology and need to be taken into account in practice, e.g., calibrating for updating biases (Benjamin, 2019), accounting for the possibility of uncertain beliefs (Enke and Graeber, 2023), dealing with the complexity that comes from explicitly spelling out the incentives of the elicitation mechanism (Danz et al., 2022) or from the need to perform contingent reasoning (Esponda and Vespa, 2014, 2023), etc. Nevertheless, addressing all these issues is outside the scope of the present paper, as they are all orthogonal to the fundamental long-standing belief identification problem that this paper tackles.

A. Proofs

Proof of Theorem 1. By (P_0) all conditional beliefs $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ are uniquely identified by any standard incentive-compatible elicitation task (e.g., a binarized scoring rule) applied K times, once for each $s \in S$. Since π_T is in the convex hull of $\{\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)\}$, there is $\lambda = (\lambda_1, \ldots, \lambda_K) \in \mathbb{R}_+^K$ with $\lambda_1 + \cdots + \lambda_K = 1$ such that

$$\pi_T = \sum_{k=1}^{K} \lambda_k \pi_T(\cdot|s_k), \tag{A.1}$$

where, by (P_2), we know π_T. By the law of total probability, π_S solves this system. Moreover, it is the unique solution if and only if matrix

$$\Pi = \begin{bmatrix} \pi_T(t_1|s_1) & \cdots & \pi_T(t_N|s_1) \\ \vdots & \ddots & \vdots \\ \pi_T(t_1|s_K) & \cdots & \pi_T(t_N|s_K) \end{bmatrix}$$

is such that $\text{rank}(\Pi) = K$. The latter holds if and only if (P_3) is satisfied. Then, by the chain rule of probability, we obtain $\pi(s,t) = \pi_S(s)\pi_T(t|s)$. Finally, by the definition of conditional probability, we have

$$\pi_S(s|E) = \frac{\pi(\{s\} \times E)}{\pi_T(E)}. \tag{A.2}$$

And, by (P_1), we obtain $\mu = \pi_S(\cdot|E)$, which completes the proof. □

Proof of Theorem 2. First, we will prove that, if u is defined as in (10) then (u, μ) is a SEU representation. Take arbitrary $(\alpha_1, \ldots, \alpha_K) \in \mathbb{R}^K$ such that $\sum_{k=1}^{K} \mu(s_k)\alpha_k = \alpha$, and $\beta > 0$. Then, for all $f \in \mathcal{F}_S$,

$$\mathbb{E}_\mu(u(f)) = \alpha + \beta \mathbb{E}_{\tilde{\mu}}(\tilde{u}(f)), \tag{A.3}$$

which completes this part of the proof.

Let us now prove the converse, i.e., if $(\tilde{u}, \mu)$ is a SEU then $\tilde{u}$ is necessarily defined as in (10). For each $k = 1, \ldots, K$, define the (convex) range of the original state-utility function, $Y_k := u_k(Q) \subseteq \mathbb{R}$. Then, it is obvious that, the state-utility function $\tilde{u}_k$ is obtained via a strictly increasing transformation of u_k, i.e., there is a continuous strictly increasing $\phi_k : Y_k \to \mathbb{R}$ such that

$$\tilde{u}_k = \phi_k \circ u_k. \tag{A.4}$$

This is because both u_k and $\tilde{u}_k$ represent the same state-preferences.

We will now show that ϕ_k is linear. Take an arbitrary $y \in \text{int}(Y_1 \times \cdots \times Y_K)$, and for each $k = 1, \ldots, K$ define $y^k \in \mathbb{R}^K$ by

$$y_\ell^k = \begin{cases} y_\ell & \text{if } \ell \neq k, \\ y_k + \delta/\mu(s_k) & \text{if } \ell = k. \end{cases} \tag{A.5}$$

Since y is an interior point, there exists some $\delta_y > 0$ such that $y^k \in Y_1 \times \cdots \times Y_K$ for all $\delta \in (0, \delta_y)$, and every $k = 1, \ldots, K$. Hence, for each $k = 1, \ldots, K$, there exists an act $f^k \in \mathcal{F}_S$ such that, for every $\ell = 1, \ldots, K$, it is the case that

$$u_\ell(f_\ell^k) = y_\ell^k, \tag{A.6}$$

Therefore, by construction, we obtain

$$\mathbb{E}_\mu(u(f^k)) = \sum_{\ell=1}^K \mu(s_\ell) y_\ell^k = \sum_{\ell=1}^K \mu(s_\ell) y_\ell + \delta. \tag{A.7}$$

Since the right hand-side does not depend on k, we have $f^1 \sim \cdots \sim f^K$. Then, since $(\tilde{u}, \mu)$ is a SEU representation, we obtain

$$\mathbb{E}_\mu(\tilde{u}(f^1) = \cdots = \mathbb{E}_\mu(\tilde{u}(f^K). \tag{A.8}$$

By combining (A.4) and (A.6), we get

$$\mathbb{E}_\mu(\tilde{u}(f^k) = \sum_{\ell=1}^K \mu(s_\ell) \phi_\ell(y_\ell^k), \tag{A.9}$$

and consequently, by (A.8), it follows that for any two distinct $k, \ell = 1, \ldots, K$,

$$\sum_{m=1}^K \mu(s_m) \phi_k(y_m^k) = \sum_{m=1}^K \mu(s_m) \phi_\ell(y_m^\ell).$$

Using the definition of y^k and y^ℓ from (A.5), the previous equation can be rewritten as

$$\mu(s_k)\phi_k(y_k + {}^{\delta}/_{\mu(s_k)}) + \mu(s_\ell)\phi_\ell(y_\ell) = \mu(s_k)\phi_k(y_k) + \mu(s_\ell)\phi_\ell(y_\ell + {}^{\delta}/_{\mu(s_\ell)}).$$

Rearranging terms, dividing both sides with δ, and taking right limits, yields

$$\phi'_{k+}(y_k) = \lim_{\delta \downarrow 0} \frac{\phi_k(y_k + {}^{\delta}/_{\mu(s_k)}) - \phi_k(y_k)}{{}^{\delta}/_{\mu(s_k)}} = \lim_{\delta \downarrow 0} \frac{\phi_\ell(y_\ell + {}^{\delta}/_{\mu(s_\ell)}) - \phi_\ell(y_\ell)}{{}^{\delta}/_{\mu(s_\ell)}} = \phi'_{\ell+}(y_\ell). \tag{A.10}$$

Note that the right derivatives are well-defined as the respective domains Y_k and Y_ℓ are convex sets. We repeat this exercise with any $y' \in \text{int}(Y_1 \times \cdots \times Y_K)$ which agrees with y at all coordinates except k, i.e., $y_k \neq y'_k$ and $y_\ell = y'_\ell$ for all $\ell \neq k$. Thus, we obtain

$$\phi'_{k+}(y'_k) = \phi'_{\ell+}(y'_\ell). \tag{A.11}$$

But, since $y_\ell = y'_\ell$, it follows directly that, for any two $y_k, y'_k \in \text{int}(Y_k)$,

$$\phi'_{k+}(y_k) = \phi'_{k+}(y'_k), \tag{A.12}$$

i.e., the right derivative of ϕ_k is constant in the interior of its domain. Therefore, together with continuity (including at the boundaries in case those belong to Y_k) it implies that ϕ_k is linear for all $k = 1, \ldots, K$. But then, by (A.10), the slope of ϕ_k is the same for all $k = 1, \ldots, K$, which completes the proof. □

Lemma A1. *There is a CSI-SEU representation $(\bar{u}, \bar{\pi})$ if and only if $\succeq$ satisfies $(A_0) - (A_5)$. Moreover, the conditional beliefs $\bar{\pi}_T(\cdot|s_1), \ldots, \bar{\pi}_T(\cdot|s_K)$ are uniquely identified from $\succeq$.*

Proof. It follows directly from applying Anscombe and Aumann (1963) to $\succeq_s$ for every $s \in S$. □

Lemma A2. *Suppose that $\succeq$ satisfies $(A_0) - (A_5)$, and let $\bar{\pi}_T(\cdot|s_1), \ldots, \bar{\pi}_T(\cdot|s_K)$ be the uniquely identified conditional beliefs from Lemma A1. Then, there is some CSI-SEU representation $(\bar{u}, \bar{\pi})$ with $\bar{\pi}_T = \pi_T^{\text{obj}}$ if and only if $\succeq$ satisfies (A_6).*

Proof. NECESSITY: By (A_0) and (A_3), there exists some $\delta > 0$, and a pair of lotteries $q_s^{\text{high}}, q_s^{\text{low}} \in Q$ for each $s \in S$, such that

$$\bar{u}_s(q_s^{\text{high}}) - \bar{u}_s(q_s^{\text{low}}) = \delta.$$

Then, take $\mathcal{E} = \{f^{\text{high}}, f^{\text{low}}\}$ such that, for every $s \in S$ and $t \in T$,

$$\begin{aligned} f_{s,t}^{\text{high}} &:= q_s^{\text{high}}, \\ f_{s,t}^{\text{low}} &:= q_s^{\text{low}}. \end{aligned}$$

Note that, for every $f \in \mathcal{F}_{\mathcal{E}}$,

$$\bar{u}_s(f_{s,t}) = \lambda_{s,t}^{\mathcal{E},f}\delta + \bar{u}_s(q_s^{\text{low}}) \tag{A.13}$$

Hence, for all $f \in \mathcal{T}_{\mathcal{E}}$, by T-measurability of $\lambda^{\mathcal{E},f}$,

$$\mathbb{E}_{\bar{\pi}}\big(\bar{u}(f)\big) = \delta \sum_{t\in T} \bar{\pi}_T(t)\lambda_t^{\mathcal{E},f} + \sum_{s\in S} \bar{\pi}_S(s)\bar{u}_s(q_s^{\text{low}}). \tag{A.14}$$

Furthermore, for the same $f \in \mathcal{T}_{\mathcal{E}}$, we obtain

$$\begin{aligned} \mathbb{E}_{\bar{\pi}}\big(\bar{u}(f^{\text{obj}})\big) &= \sum_{s\in S} \bar{\pi}_S(s)\bar{u}_s(f_s^{\text{obj}}) && \text{(A.15)} \\ &= \sum_{s\in S} \bar{\pi}_S(s) \sum_{t\in T} \bar{\pi}_T(t)\bar{u}_s(f_{s,t}) && \text{(A.16)} \\ &= \delta \sum_{t\in T} \bar{\pi}_T(t)\lambda_t^{\mathcal{E},f} + \sum_{s\in S} \bar{\pi}_S(s)\bar{u}_s(q_s^{\text{low}}). && \text{(A.17)} \end{aligned}$$

where (A.15) follows from S-measurability of f^{obj}, (A.16) follows from the definition of f^{obj} combined with $\bar{\pi}_T = \pi_T^{\text{obj}}$, and (A.17) follows from (A.13) combined with T-measurability of $\lambda^{\mathcal{E},f}$. Hence, $f \sim f^{\text{obj}}$, i.e., (A_6) holds.

SUFFICIENCY: Let $\mathcal{E} = \{f^{\text{high}}, f^{\text{low}}\}$ be the menu of S-measurable acts such that $f \sim f^{\text{obj}}$ for all $f \in \mathcal{F}_{\mathcal{E}}$. Take an arbitrary CSI-SEU $(\hat{v}, \hat{\pi})$ of $\succeq$. Note that there exist $\alpha_s \in \mathbb{R}$ and $\beta_s > 0$ such that $\alpha_s + \beta_s\hat{u}_s(f_s^{\text{high}}) = 1$ and $\alpha_s + \beta_s\hat{u}_s(f_s^{\text{low}}) = 0$. Then, define the normalized S-measurable $\bar{u}$ by

$$\bar{u}_s := \alpha_s + \beta_s\hat{u}_s \tag{A.18}$$

for each $s \in S$. Moreover, for every $s \in S$, define the new marginal belief

$$\bar{\pi}_S(s) := \frac{\hat{\pi}_S(s)/\beta_s}{\sum_{s'\in S} \hat{\pi}_S(s')/\beta_{s'}}, \tag{A.19}$$

and observe that for every $f \in \mathcal{F}$, we obtain

$$\mathbb{E}_{\hat{\pi}}\big(\hat{u}(f)\big) = \alpha + \beta\mathbb{E}_{\bar{\pi}}\big(\bar{u}(f)\big), \tag{A.20}$$

where $\alpha := \sum_{s\in S} \alpha_s\bar{\pi}_S(s)$ and $\beta := \sum_{s'\in S} \hat{\pi}_S(s')/\beta_{s'} > 0$ are both constants, and the joint belief $\bar{\pi}$ is defined by

$$\bar{\pi}(s,t) := \bar{\pi}_S(s)\hat{\pi}_T(t|s). \tag{A.21}$$

Hence, the pair $(\bar{v}, \bar{\pi})$ is a CSI-SEU.

Fix an arbitrary $t \in T$, and take the act

$$\langle t \rangle := f_{S\times\{t\}}^{\text{high}} f^{\text{low}},$$

meaning that for every state (s,t), the state-utility becomes

$$\bar{u}_s(\langle t \rangle_{s,t'}) = \begin{cases} 1 & \text{if } t = t', \\ 0 & \text{if } t \neq t'. \end{cases}$$

Hence, by construction, we have

$$\mathbb{E}_{\bar{\pi}}(\bar{u}(\langle t\rangle)) = \sum_{s\in S}\bar{\pi}_S(s)\bar{\pi}_T(t|s) = \bar{\pi}_T(t). \tag{A.22}$$

Moreover, by the definition of $\langle t\rangle^{\text{obj}}$, we obtain

$$\mathbb{E}_{\bar{\pi}}(\bar{u}(\langle t\rangle^{\text{obj}})) = \sum_{s\in S}\bar{\pi}_S(s)\pi_T^{\text{obj}}(t) = \pi_T^{\text{obj}}(t). \tag{A.23}$$

Finally, note that by construction, the act $\langle t\rangle$ belongs to $\mathcal{T}_{\mathcal{E}}$. Hence, by (A_6), we have $\bar{\pi}_T = \pi_T^{\text{obj}}$. □

Lemma A3. *Suppose that $\succeq$ satisfies $(A_0)-(A_5)$, and let $\bar{\pi}_T(\cdot|s_1),\ldots,\bar{\pi}_T(\cdot|s_K)$ be the uniquely identified conditional beliefs from Lemma A1. Then, $\bar{\pi}_T(\cdot|s_1),\ldots,\bar{\pi}_T(\cdot|s_K)$ are linearly independent if and only if $\succeq$ satisfies (A_7).*

Proof. Intermediate step: Fix an arbitrary menu of S-measurable acts $\mathcal{E} = \{f^{\text{high}}, f^{\text{low}}\}$ such that $f^{\text{high}} \succ_{s,t} f^{\text{low}}$ for all states (s,t). Take a CSI-SEU representation $(\hat{v},\hat{\pi})$ of $\succeq$, and like in the proof of sufficiency in the previous lemma, obtain another CSI-SEU representation $(\bar{v},\bar{\pi})$ such that $\bar{u}_s(f_s^{\text{high}}) = 1$ and $\bar{u}_s(f_s^{\text{low}}) = 1$. Hence, for every $f \in \mathcal{F}_{\mathcal{E}}$, and every state (s,t), we have

$$\bar{u}_s(f_{s,t}) = \lambda_{s,t}^{\mathcal{E},f}.$$

Hence, for every $f\in\mathcal{T}_{\mathcal{E}}$, by T-measurability of $\lambda^{\mathcal{E},f}$, we get

$$\mathbb{E}_{\bar{\pi}}\big(\bar{u}(f)\big) = \sum_{t\in T}\lambda_t^{\mathcal{E},f}\sum_{s\in S}\bar{\pi}_S(s)\bar{\pi}_T(t|s). \tag{A.24}$$

Sufficiency: Suppose that $\bar{\pi}_T(\cdot|s_1),\ldots,\bar{\pi}_T(\cdot|s_K)$ are not linearly independent. Hence, there exists some $\bar{\pi}'_S \in \Delta(S)$ with $\bar{\pi}'_S \neq \bar{\pi}_S$ such that

$$\sum_{s\in S}\bar{\pi}'_S(s)\bar{\pi}_T(\cdot|s) = \sum_{s\in S}\bar{\pi}_S(s)\bar{\pi}_T(\cdot|s) = \bar{\pi}_T. \tag{A.25}$$

Consider the preference relation $\succeq'$ which is represented by $(\bar{v},\bar{\pi}')$. By S-measurability of $\bar{u}$, it follows that $\succeq'$ satisfies $(A_0)-(A_5)$. Moreover, by (A.24), it follows that $\mathbb{E}_{\bar{\pi}}\big(\bar{u}(f)\big) = \mathbb{E}_{\bar{\pi}'}\big(\bar{u}(f)\big)$ for every $f\in\mathcal{T}_{\mathcal{E}}$, i.e.,

$$\succeq\; = \;\succeq' \text{ in } \mathcal{T}_{\mathcal{E}}. \tag{A.26}$$

For any $\lambda_0,\lambda_1,\lambda_2\in(0,1)$, define the following acts that belong to $\mathcal{F}_{\mathcal{E}}$:

$$\begin{aligned} f^0 &:= \lambda_0 f^{\text{high}} + (1-\lambda_0)f^{\text{low}},\\ f^1 &:= \lambda_1 f^{\text{high}} + (1-\lambda_1)f^{\text{low}},\\ f^2 &:= \lambda_2 f^{\text{high}} + (1-\lambda_2)f^{\text{low}}. \end{aligned}$$

Then, observe that the following equivalences hold for all $s\in S$:

$$\begin{aligned} f^1_{\{s\}\times T}f^2 \succeq f^0 &\Leftrightarrow \bar{\pi}_S(s)\lambda_1 + (1-\bar{\pi}_S(s))\lambda_2 \geq \lambda_0,\\ f^1_{\{s\}\times T}f^2 \succeq' f^0 &\Leftrightarrow \bar{\pi}'_S(s)\lambda_1 + (1-\bar{\pi}'_S(s))\lambda_2 \geq \lambda_0. \end{aligned}$$

By $\bar{\pi}_S \neq \bar{\pi}'_S$, there is some $s\in S$ such that $\bar{\pi}_S(s) > \bar{\pi}'_S(s)$. Hence, for some $0<\lambda_2<\lambda_0<\lambda_1<1$, we obtain $f^1_{\{s\}\times T}f^2 \succeq f^0$ and $f^1_{\{s\}\times T}f^2 \not\succeq' f^0$, meaning that

$$\succeq\; \neq \;\succeq' \text{ in } \mathcal{F}_{\mathcal{E}}. \tag{A.27}$$

Hence, $\succeq$ is not uniquely extended from $\mathcal{T}_{\mathcal{E}}$ to $\mathcal{F}_{\mathcal{E}}$, meaning that (A_6) is violated.

Necessity: Suppose that $\bar{\pi}_T(\cdot|s_1), \ldots, \bar{\pi}_T(\cdot|s_K)$ are linearly independent. Take an arbitrary $\succeq'$ satisfying $(A_0) - (A_5)$, such that $\succeq = \succeq'$ in $\mathcal{T}_{\mathcal{E}}$. This implies that $\succeq_s = \succeq'_s$ for all $s \in S$, as $\mathcal{T}_{\mathcal{E}}$ places restrictions only across T. Therefore, the CSI-SEU representations $(\bar{u}, \bar{\pi})$ and $(\bar{u}', \bar{\pi}')$ that we constructed in the intermediate step, are such that $\bar{u} = \bar{u}'$ and $\bar{\pi}_T(\cdot|s) = \bar{\pi}'_T(\cdot|s)$ for all $s \in S$.

Suppose that $\bar{\pi}_S \neq \bar{\pi}'_S$. Then, by linear independence, there exists some $t \in T$ such that

$$\bar{\pi}'_T(t) = \sum_{s \in S} \bar{\pi}'_S(s)\bar{\pi}_T(t|s) < \sum_{s \in S} \bar{\pi}_S(s)\bar{\pi}_T(t|s) = \bar{\pi}_T(t). \tag{A.28}$$

Pick some $\lambda \in (0,1)$ such that $\bar{\pi}'_T(t) < \lambda < \bar{\pi}_T(t)$, and define the act h such that $\bar{u}_{s,t}(h_{s,t}) = \lambda$ for every (s,t). It is not difficult to verify that such act exists and $h \in \mathcal{T}_{\mathcal{E}}$. Moreover, we have $f^{\text{high}}_{S\times\{t\}} f^{\text{low}} \in \mathcal{T}_{\mathcal{E}}$. Hence, by (A.24), we obtain

$$f^{\text{high}}_{S\times\{t\}} f^{\text{low}} \succeq h \text{ and } f^{\text{high}}_{S\times\{t\}} f^{\text{low}} \not\succeq' h, \tag{A.29}$$

which contradict $\succeq = \succeq'$ in $\mathcal{T}_{\mathcal{E}}$. Therefore, it must necessarily be the case that $\bar{\pi}_S = \bar{\pi}'_S$. But then, this implies $\bar{\pi} = \bar{\pi}'$, which together with $\bar{u} = \bar{u}'$, yields

$$\succeq = \succeq' \text{ in } \mathcal{F}_{\mathcal{E}}, \tag{A.30}$$

and the proof is complete. □

Proof of Theorem 3. The proof follows directly from Lemmas A1, A2 and A3. □

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