Belief identification by proxy

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Roadmap

1. The problem
2. My solution
3. Proof of concept
4. Concluding

Roadmap

Background

We want to identify beliefs

Why?

1. Make in-sample predictions:
   - Investor's belief about an asset explains her investment on this asset
2. Make out-of-sample predictions:
   - Investor's belief about an asset explains her overall investment behavior
   - Investor's belief about an asset explains investment behavior of others
3. Use beliefs for making decisions:
   - Political campaign makes strategic decision based on election forecasts
4. Compare beliefs:
   - Assess expertise of professional forecasters based on their accuracy
   - Compare opinions of Democrats and Republicans to measure polarization
5. Aggregate opinions:
   - Substitute polls on intended vote with polls on forecasted outcome

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How can we identify beliefs?

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Big question

How can we identify beliefs?

Standard answer

Betting behavior reveals beliefs

Wife's insurance problem (Aumann, 1971)

- Husband suffers from Guillain-Barre syndrom
- His risk-neutral wife is offered insurance package

	recovers (s_1)	paralyzed (s_2)	Expected Utility
insurance	\$0	\$10k	$10\bar{\mu}_2$
no insurance	\$1k	\$1k	$\bar{\mu}_1 + \bar{\mu}_2$

- Observed choice data: Wife is indifferent between two acts
- Wife's belief identified: $\bar{\mu}_1 = 90\%$

The identification problem (Drèze, 1961)

- Previously we assumed state-independent SEU model:

$$\mathbb{E}_{\bar{\mu}}(\bar{u}(x_1, x_2)) = \bar{\mu}_1 \underbrace{x_1}_{\bar{u}(x_1)} + \bar{\mu}_2 \underbrace{x_2}_{\bar{u}(x_2)}$$

- Take alternative state-dependent utility SEU model:

$$\mathbb{E}_{\mu}(u(x_1, x_2)) = \mu_1 \underbrace{\frac{\bar{\mu}_1}{\mu_1} x_1}_{u_1(x_1)} + \mu_2 \underbrace{\frac{\bar{\mu}_2}{\mu_2} x_2}_{u_2(x_2)}$$

- The two models represent the same preferences
- Nonetheless, they involve different belief!!! (Identification problem)
- Important remark: The identification problem arises even when there is a state-independent SEU (Savage ,1954; Anscombe & Aumann, 1963)!

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2. How bad is it to assume state-independence?

   It depends...

How bad is it to assume state-independence?

Traditional view

Not so bad!!!

- It is irrelevant if beliefs actually exist outside the model
- We want a model that:
  - disentangles beliefs from utilities, in order to provide foundations of subjective probability
  - makes in-sample predictions
- The job is done by both models
- We choose the state-independent model because it is simpler.

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Modern view

Not that good!!!

- Beliefs are unobservable primitive
- We also care about:
  - out-of-sample predictions
  - using beliefs for decisions
  - comparing beliefs
  - aggregating opinions
- We need to choose the model that involves the actual belief
- The state-independent model is the "correct one" only if the agent has no stakes in the event!!!
- A state-independent model does not always exist!

Literature: What does theory say so far?

> *"the problem is serious, but I am willing to live with it until something better comes along"*
>
> *Leonard J. Savage (1971)*
> *letter correspondence with Bob Aumann*

- Go beyond traditional betting data:
  - Dréze (1961): agent can influence the state realization
  - Fishburn (1973); Karni (1992, 1993): agent makes choices conditional on different events
  - Karni, Schmeidler & Vind (1983): choices given hypothetical beliefs
  - Schervish, Seidenfeld & Kadane (1990): agent compares lotteries at different states
  - Lu (2019): agent updates beliefs using information that analyst provides
- No consensus on one of these solutions
  - Set of applications is very narrow
  - Implementation is very complex
- The problem is very difficult, and still open!!!

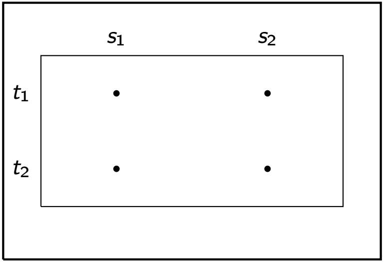
Roadmap

My approach: A variant of the strategy method

Main idea:

Keep using betting data, albeit over an extended state space.

- Introduce a proxy variable: $T = \{t_1, t_2\}$



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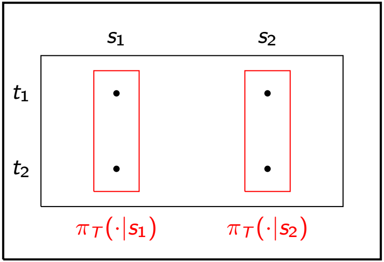

- Instead of eliciting directly beliefs about S,

  elicit beliefs about T conditional on each realization of S.

What is a proxy?

Definition

We say that T is a proxy for S, whenever the following are satisfied:

(P_0) **No stakes:** Given each realization of S, the agent has no stakes in the proxy

- The actual belief $\pi_T(\cdot|s)$ is the one given by the conditional SI-SEU representation

(P_1) **Objective marginal:** The marginal π_T is known

- There is an exogenously given π_T^{obj} such that $\pi_T = \pi_T^{\text{obj}}$

(P_2) **Uninformative event:** There is some subset $E \subseteq T$ such that $\mu = \pi_S(\cdot|E)$

(P_3) **Linear independence:** $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ are linearly independent

- With two states, this assumption reduces to correlation

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Example 1

We stochastically influence the realization of the state space.

Stochastic intervention

What probability does the wife attach to her husband recovering?

$$S = \{\text{husband recovers } (s_1),\ \text{husband paralyzed } (s_2)\}$$
$$T = \{\text{treatment group } (t_1),\ \text{control group } (t_2)\}$$

(P_0) **No stakes:** Given health outcome, wife does not care whether the husband received the drug or the placebo

(P_1) **Objective marginal:** Known chances to be placed in placebo group

(P_2) **Uninformative event:** Placebo has no effect on recovery

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	s_1	s_2
t_1	•	•
t_2	•	•

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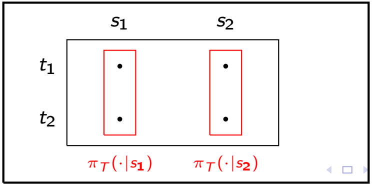

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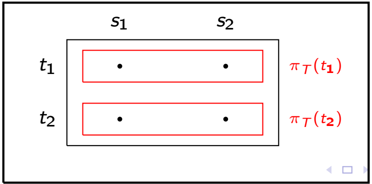

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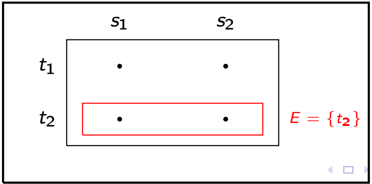

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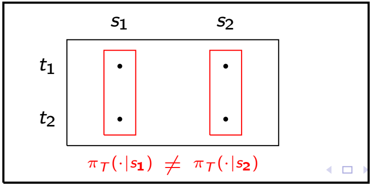

Example 2

We provide evidence which is either true or fabricated.

Evidence with stochastic informativeness

What probability does the wife attach to her husband recovering?

$$S = \{\text{husband recovers } (s_1),\ \text{husband paralyzed } (s_2)\}$$
$$T = \{\text{expert's opinion } (t_1),\ \text{charlatan's opinion } (t_2)\}$$

(P_0) **No stakes:** Given health outcome, wife does not care about where the opinion came from

(P_1) **Objective marginal:** Known chances of opinion coming from expert

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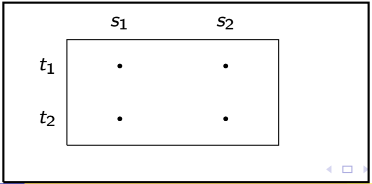

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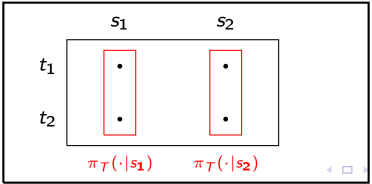

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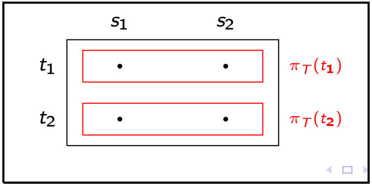

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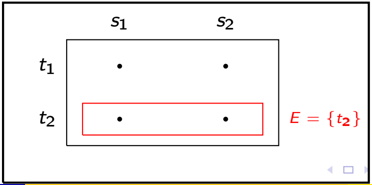

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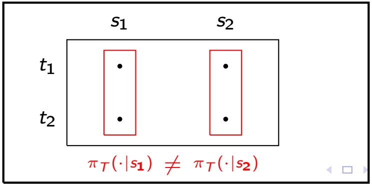

Example 3

We partition the population based on some demographic.

Population partition

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$$T = \{\text{gene } (t_1),\ \text{no gene } (t_2)\}$$

(P_0) **No stakes:** Given health outcome, gene is irrelevant
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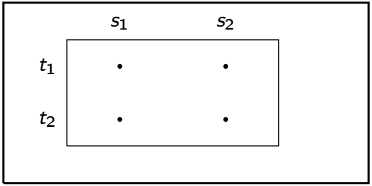

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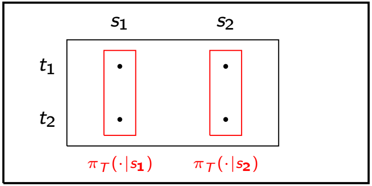

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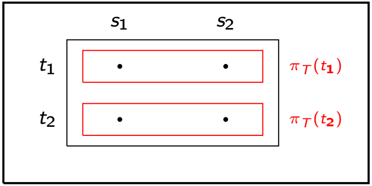

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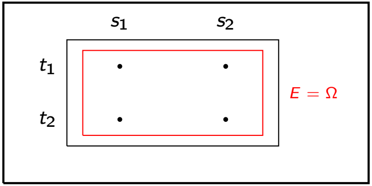

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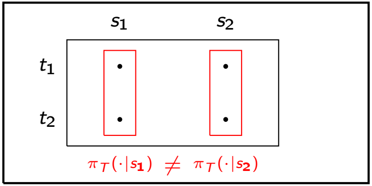

Main result

If there is a proxy, beliefs about original variable are identified.

Identification Theorem

Theorem (Identification of beliefs)

Suppose that T satisfies:

(P_0) ***No stakes:*** *The agent has no stakes in T given S*

(P_1) ***Objective marginal:*** π_T *is known*

(P_2) ***Uninformative event:*** $\mu = \pi_S(\cdot|E)$ *for some* $E \subseteq T$

Then, μ is identified with traditional choice data if and only if T satisfies

(P_3) ***Linear independence:*** $\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ *linearly independent*

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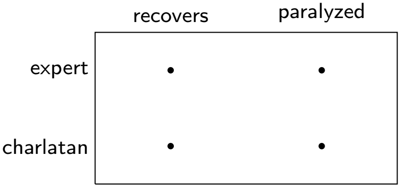

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Identify the joint belief π

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	recovers	paralyzed
expert	•	•
charlatan	1/3	2/3

Conditional belief with respect to E

Identification Theorem: Relationship to IV's

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  - Econometrics: Orthogonality
  - Decision Theory: No stakes about S
- Replaced with other exogenous assumptions that are easy to justify
  - Econometrics: Exclusion criterion
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Identification Theorem: Exogeneity of assumptions

Theorem (Identification of beliefs)

Suppose that T satisfies:

(P_0) ***No stakes:*** *The agent has no stakes in T given S*

(P_1) ***Objective marginal:*** *π_T is known*

(P_2) ***Uninformative event:*** *$\mu = \pi_S(\cdot|E)$ for some $E \subseteq T$*

Then, μ is identified if and only if T satisfies

(P_3) ***Linear independence:*** *$\pi_T(\cdot|s_1), \ldots, \pi_T(\cdot|s_K)$ linearly independent*

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Roadmap

Proof of concept: Does the method work?

Hypothesis:

Beliefs elicited directly = Beliefs identified by proxy

if and only if

The subjects do not have stakes in the underlying state space

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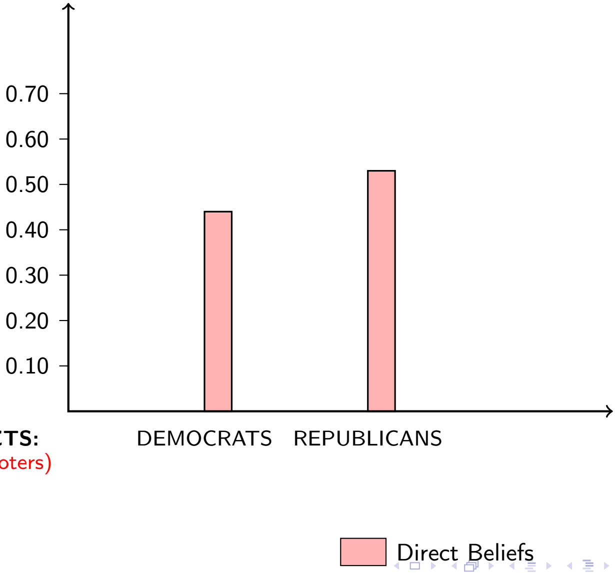
- Indirect identification (via my method):
  - Among those liking X, what do you think is the percentage of men?
  - Among those disliking X, what do you think is the percentage of men?

What percentage of voters like Trump?



SUBJECTS:
(not the voters)

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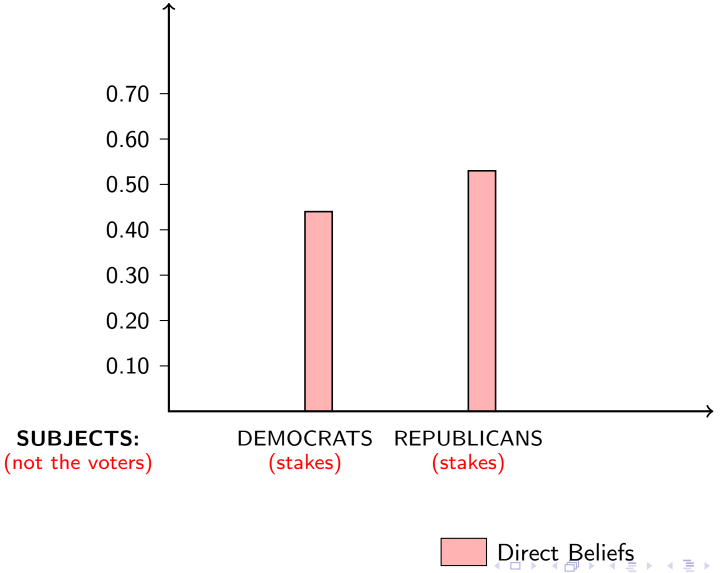

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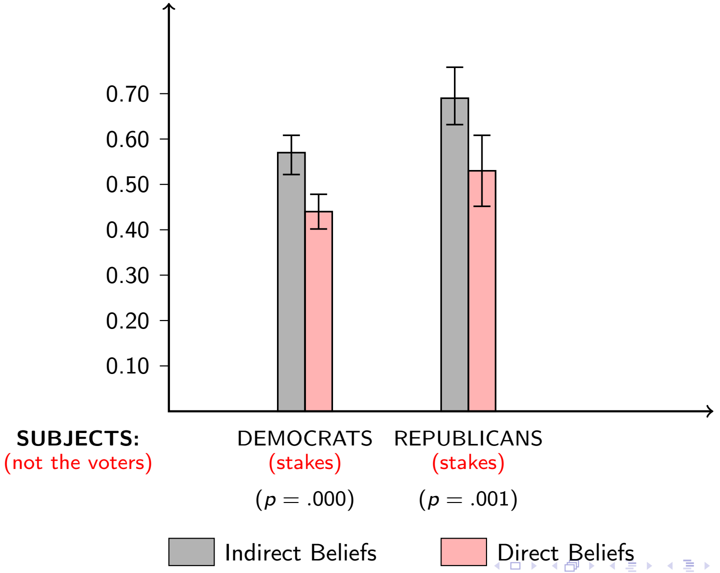

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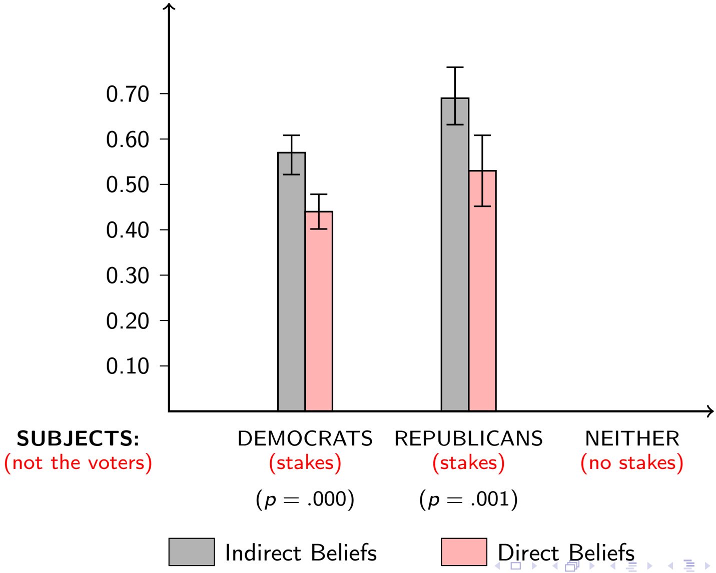

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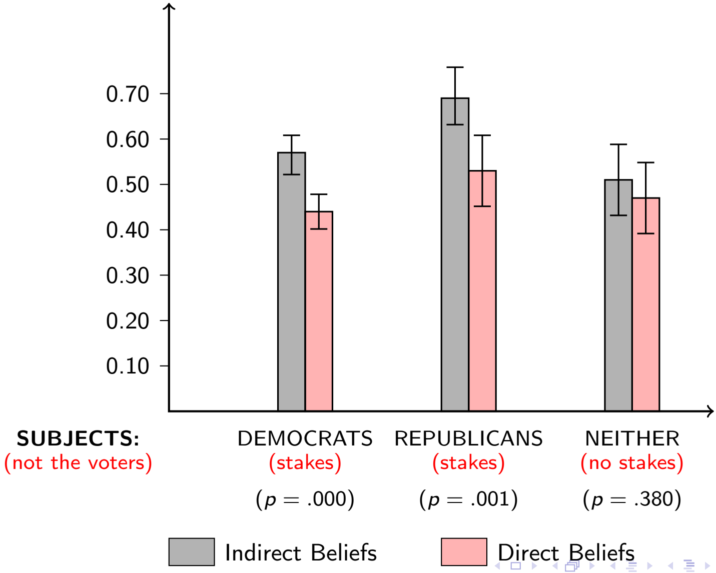
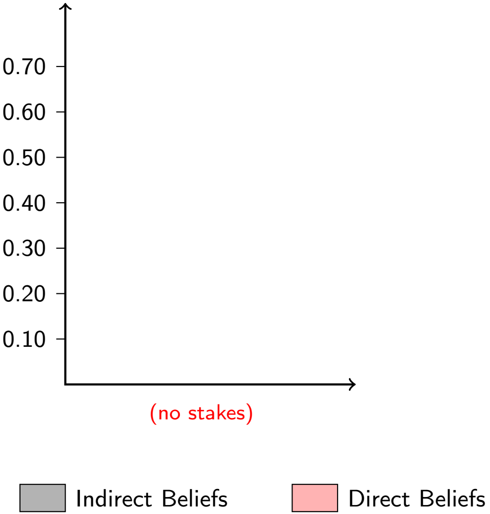


What percentage of people like rock better than hip hop?



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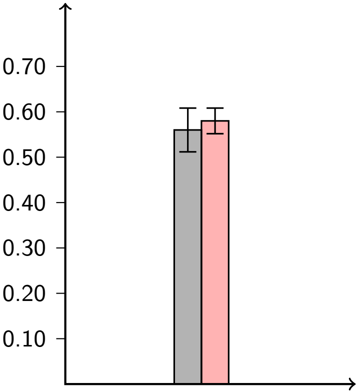

(no stakes)

($p = .698$)

Indirect Beliefs   Direct Beliefs

Roadmap

Take-home message

- Theoretically: simple solution to long-standing problem!!!
  - Identification result holds for any finite state space
  - Decision-theoretic foundations
  - Definition of actual utility
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Thanks for listening!!!