

Type space models vs. state space models

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EpiCenter Spring Course on Epistemic Game Theory
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Roadmap

- 1 State space model
- 2 Belief hierarchies
- 3 Equivalence of representations of belief hierarchies

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 - $P_1.$ $\omega \in P_i(\omega)$
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 - A \mathcal{P}_i -measurable function $c_i : \Omega \rightarrow C_i$, i.e., $c_i(\omega) = c_i(\omega')$ for all $\omega' \in P_i(\omega)$.
 - A prior belief $\pi_i \in \Delta(\Omega)$ such that $\pi_i(P_i(\omega)) > 0$ for every $\omega \in \Omega$.
If $\pi_i = \pi$ for all $i \in I$, then we say that there is a common prior.

Posterior beliefs

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Posterior beliefs

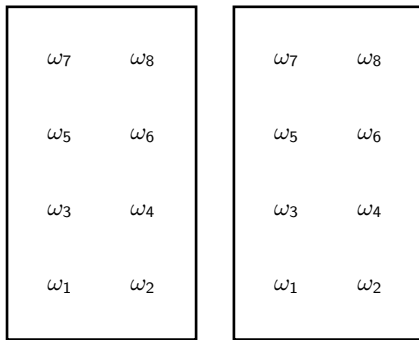
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- Player i is certain about her own information, i.e., $\pi_i(P_i(\omega)|P_i(\omega)) = 1$.
- Moreover, i is certain about her own choice at ω , i.e.,

$$\begin{aligned}
 \pi_i(c_i^{-1}(\omega)|P_i(\omega)) &= \pi_i(\{\omega' \in \Omega : c_i(\omega') = c_i(\omega)\}|P_i(\omega)) \\
 &\geq \pi_i(P_i(\omega)|P_i(\omega)) \\
 &= 1
 \end{aligned}$$

An example



- State space $\Omega = \{\omega_1, \dots, \omega_8\}$

An example

1/8	0	2/16	2/16
1/8	0	1/16	3/16
1/8	0	3/16	1/16
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- Prior beliefs $\pi_i \in \Delta(\Omega)$
- Information partitions \mathcal{P}_i

An example

1/4	0
1/4	0
1/4	0
1/4	1

1/2	1/2
1/4	3/4
3/4	1/4
1	0

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An example

1/4	0
1/4	0
<i>L</i>	<i>H</i>
1/4	0
1/4	1

1/2	<i>L</i>	1/2
1/4	<i>L</i>	3/4
3/4	<i>H</i>	1/4
1	<i>L</i>	0

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- Prior beliefs $\pi_i \in \Delta(\Omega)$
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- Posterior beliefs $\pi_i(\cdot | P_i(\omega)) \in \Delta(\Omega)$
- Choice functions $c_i : \Omega \rightarrow C_i$

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Belief hierarchies revisited

- Consider the following sequence:

$$\begin{aligned}
 \Theta_i^0 &:= C_j \\
 \Theta_i^1 &:= \Theta_i^0 \times \Delta(\Theta_j^0) \\
 &\vdots \\
 \Theta_i^k &:= \Theta_i^{k-1} \times \Delta(\Theta_j^{k-1}) \\
 &\vdots
 \end{aligned}$$

- A belief hierarchy of player i is a sequence of probability measures $(\pi_i^1, \pi_i^2, \dots) \in \Delta(\Theta_i^0) \times \Delta(\Theta_i^1) \times \dots$.

From states to belief hierarchies

Each state $\omega \in \Omega$ is uniquely associated with a belief hierarchy,
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$$\pi_i^1(\omega)(B_0) := \pi_i(\{(\omega' \in \Omega : c_j(\omega') \in B_0\} | P_i(\omega))$$

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- $\pi_i^2(\omega) \in \Delta(\Theta_i^1)$: For each (Borel) event $B_1 \subseteq \Theta_i^1$,

$$\pi_i^2(\omega)(B_1) := \pi_i(\{\omega' \in \Omega : (c_j(\omega'), \pi_j^1(\omega')) \in B_1\} | P_i(\omega))$$

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Each state $\omega \in \Omega$ is uniquely associated with a belief hierarchy, $(\pi_i^1(\omega), \pi_i^2(\omega), \dots)$:

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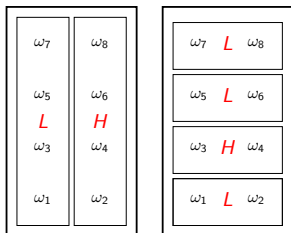
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- $\pi_i^{k+1}(\omega) \in \Delta(\Theta_i^k)$: For each (Borel) event $B_k \subseteq \Theta_i^k$,

$$\pi_i^{k+1}(\omega)(B_k) := \pi_i(\{\omega' \in \Omega : (c_j(\omega'), \pi_j^1(\omega'), \dots, \pi_j^k(\omega')) \in B_k\} | P_i(\omega))$$

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An example

1/4	0	1/2 <i>L</i> 1/2
1/4	0	1/4 <i>L</i> 3/4
<i>L</i>	<i>H</i>	3/4 <i>H</i> 1/4
1/4	0	1 <i>L</i> 0
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- Ann's first order beliefs at ω_1 :

$$\pi_a^1(\omega_1)(L) = \pi_a(\{\omega_1, \omega_2, \omega_5, \omega_6, \omega_7, \omega_8\} | P_a(\omega_1)) = 3/4.$$

An example

1/4	0	1/2 <i>L</i> 1/2
1/4	0	1/4 <i>L</i> 3/4
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- Bob's first order beliefs at ω_5 :

$$\pi_b^1(\omega_5)(H) = \pi_b(\{\omega_2, \omega_4, \omega_6, \omega_8\} | P_b(\omega_5)) = 3/4.$$

An example

1/4	0	1/2 <i>L</i> 1/2
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- Ann's second order beliefs at ω_1 :

$$\pi_a^2(\omega_1)(L, \pi_b^1) = \pi_a(\{\omega_5\} | P_a(\omega_1)) = 1/4.$$

Belief hierarchies and information sets

Proposition

The function $(\pi_i^1(\cdot), \pi_i^2(\cdot) \dots) : \Omega \rightarrow \Delta(\Theta_i^1) \times \Delta(\Theta_i^2) \times \dots$ is \mathcal{P}_i -measurable.

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Proof.

It suffices to show that

$$(\pi_i^1(\omega), \pi_i^2(\omega) \dots) = (\pi_i^1(\omega'), \pi_i^2(\omega') \dots)$$

for all $\omega' \in P_i(\omega)$. This follows directly from the fact that $\pi(\cdot | P_i(\omega)) = \pi_i(\cdot | P_i(\omega'))$ for all $\omega' \in P_i(\omega)$. □

Coherency and common certainty in coherency

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For every $\omega \in \Omega$, it is the case that $(\pi_i^1(\omega), \pi_i^2(\omega) \dots) \in H_i^$.*

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Proof.

- ① We prove that $(\pi_i^1(\omega), \pi_i^2(\omega), \dots) \in H_i^c$ for every $\omega \in \Omega$. For arbitrary $k \geq 0$ and Borel $B_k \subseteq \Theta_i^k$,

$$\begin{aligned}
 & \text{marg}_{\Theta_i^k} \pi_i^{k+2}(\omega)(B_k) \\
 = & \pi_i^{k+2}(\omega)(B_k \times \Delta(\Theta_j^k)) \\
 = & \pi_i(\{\omega' : (c_j(\omega'), \pi_j^1(\omega'), \dots, \pi_j^{k+1}(\omega')) \in B_k \times \Delta(\Theta_j^k)\} | P_i(\omega)) \\
 = & \pi_i(\{\omega' : (c_j(\omega'), \pi_j^1(\omega'), \dots, \pi_j^k(\omega')) \in B_k | P_i(\omega)) \\
 = & \pi_i^{k+1}(\omega)(B_k).
 \end{aligned}$$



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- 2 We prove that $(\pi_i^1(\omega), \pi_i^2(\omega), \dots) \in H_i^1$ for all $\omega \in \Omega$:

$$\begin{aligned} & \pi_i^{k+1}(\omega)(C_j \times \text{proj}_{\times_{\ell=0}^{k-1} \Delta(\Theta_j^\ell)} H_j^c) \\ = & \pi_i(\{\omega' : (\pi_j^1(\omega'), \dots, \pi_j^k(\omega')) \in \text{proj}_{\times_{\ell=0}^{k-1} \Delta(\Theta_j^\ell)} H_j^c \mid P_i(\omega)\}) \\ = & \pi_i(\Omega \mid P_i(\omega)) = 1. \end{aligned}$$



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- 1 $(\pi_i^1(\omega), \pi_i^2(\omega), \dots) \in H_i^c$ for every $\omega \in \Omega$.
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- 2 $(\pi_i^1(\omega), \pi_i^2(\omega), \dots) \in H_i^1$ for every $\omega \in \Omega$.
- 3 Finally, we prove by induction on m (following exactly the same steps as above) that $(\pi_i^1(\omega), \pi_i^2(\omega), \dots) \in H_i^m$ for every $\omega \in \Omega$. Hence, it follows directly that

$$(\pi_i^1(\omega), \pi_i^2(\omega), \dots) \in H_i^*$$

for all $\omega \in \Omega$.



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- **Question:** Are the two models equivalent?
- In other words, can we find a **belief-hierarchy-preserving** function taking us from one model to the other?
- The answer is yes.

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 - $c_i(\omega) := \text{proj}_{C_i} \omega$ and $[c_i] := \{\omega \in \Omega : c_i(\omega) = c_i\}$
 - $t_i(\omega) := \text{proj}_{T_i} \omega$ and $[t_i] := \{\omega \in \Omega : t_i(\omega) = t_i\}$

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An information set is a choice-type pair

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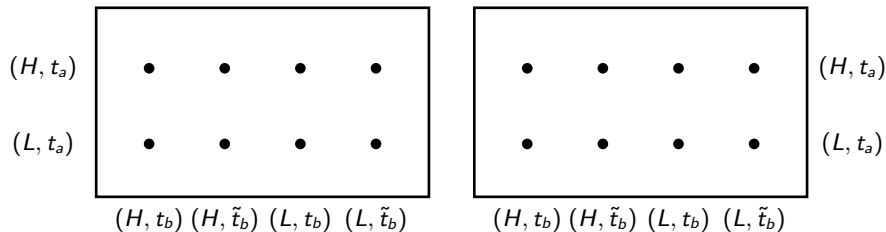
- Posterior beliefs $\pi_i(\cdot | P_i(\omega)) \in \Delta(\Omega)$ such that for each $E \subseteq \Omega$,

$$\pi_i(E | P_i(\omega)) := b_i(t_i(\omega))(\{(c_j, t_j) \in C_j \times T_j : (c_i(\omega), c_j, t_i(\omega), t_j) \in E\})$$

The posterior beliefs are the beliefs about the opponent's choice-type pairs. A prior can be constructed by taking a weighted average of the posteriors (not unique).

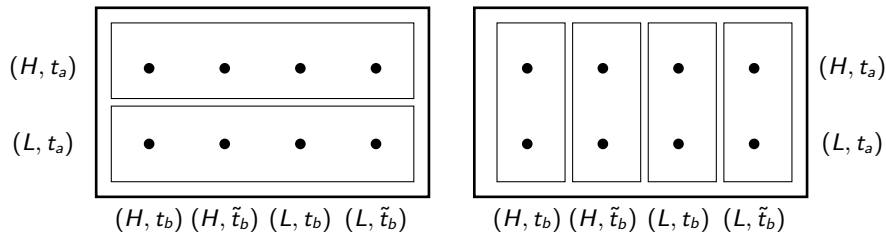
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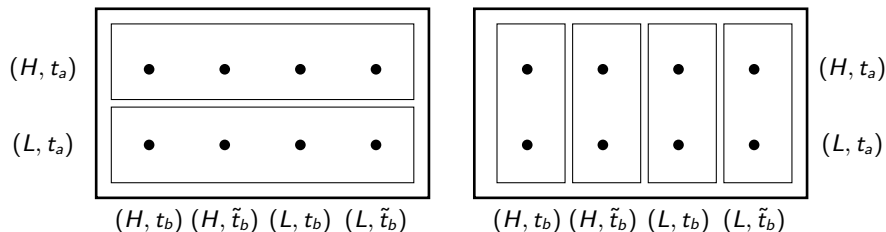
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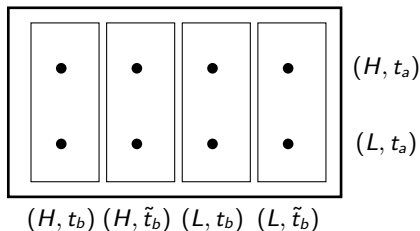
- $C_i = \{H, L\}$, $T_a = \{t_a\}$, $T_b = \{t_b, \tilde{t}_b\}$
- $b_a(t_a) = (\frac{1}{4} \otimes (H, t_b); \frac{1}{4} \otimes (H, \tilde{t}_b); \frac{1}{2} \otimes (L, \tilde{t}_b))$



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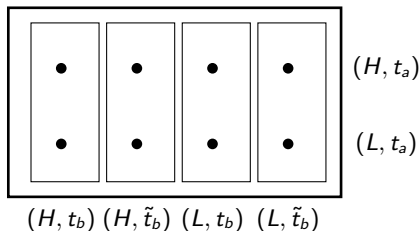
(H, t_a)	1/4	1/4	0	1/2
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- $b_b(t_b) = (\frac{1}{3} \otimes (H, t_a); \frac{2}{3} \otimes (L, t_a))$
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(H, t_a)	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border: 1px solid black; padding: 5px;">1/4</td> <td style="border: 1px solid black; padding: 5px;">1/4</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">1/2</td> </tr> </table>	1/4	1/4	0	1/2	
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(L, t_a)	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border: 1px solid black; padding: 5px;">1/4</td> <td style="border: 1px solid black; padding: 5px;">1/4</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">1/2</td> </tr> </table>	1/4	1/4	0	1/2	
1/4	1/4	0	1/2			
	(H, t_b) (H, \tilde{t}_b) (L, t_b) (L, \tilde{t}_b)					

	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border: 1px solid black; padding: 5px;">1/3</td> <td style="border: 1px solid black; padding: 5px;">1/4</td> <td style="border: 1px solid black; padding: 5px;">1/3</td> <td style="border: 1px solid black; padding: 5px;">1/4</td> </tr> </table>	1/3	1/4	1/3	1/4	(H, t_a)
1/3	1/4	1/3	1/4			
	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border: 1px solid black; padding: 5px;">2/3</td> <td style="border: 1px solid black; padding: 5px;">3/4</td> <td style="border: 1px solid black; padding: 5px;">2/3</td> <td style="border: 1px solid black; padding: 5px;">3/4</td> </tr> </table>	2/3	3/4	2/3	3/4	(L, t_a)
2/3	3/4	2/3	3/4			
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From a type space to a state space

Proposition

For each $\omega \in \Omega$ it is the case that

$$(\pi_i^1(\omega), \pi_i^2(\omega), \dots) = (\pi_i^1(t_i(\omega)), \pi_i^2(t_i(\omega)), \dots).$$

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Proof.

The proof follows directly by construction of the posterior beliefs in the state space model. □

From a state space to a type space

We begin with a state space model:

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- Fix a (finite) set T_i with the same cardinality as \mathcal{P}_i . Then, define a \mathcal{P}_i -measurable surjective function $f_i : \Omega \rightarrow T_i$, such that

$$f_i(\omega) = f_i(\omega') \text{ if and only if } P_i(\omega) = P_i(\omega').$$

Let $f_i^{-1}(t_i) := \{\omega \in \Omega : f_i(\omega) = t_i\} \in \mathcal{P}_i$.

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- Define $b_i : T_i \rightarrow \Delta(C_j \times T_j)$ by

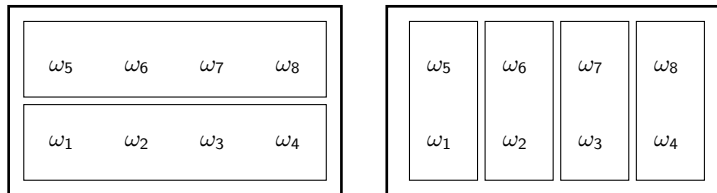
$$b_i(t_i)(c_j, t_j) := \pi_i(\{\omega' \in \Omega : (c_j(\omega'), f_j(\omega')) = (c_j, t_j)\} | f_i^{-1}(t_i))$$

From a state space to a type space

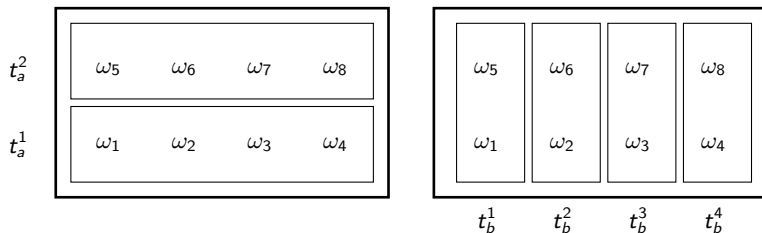
$1/4$	$1/4$	H	0	$1/2$
$1/4$	$1/4$	L	0	$1/2$

$1/3$	$1/4$	$1/3$	$1/4$
H	H	L	L
$2/3$	$3/4$	$2/3$	$3/4$

From a state space to a type space



From a state space to a type space



- $f_a(\omega_1) = f_a(\omega_2) = f_a(\omega_3) = f_a(\omega_4) = t_a^1$
 $f_a(\omega_5) = f_a(\omega_6) = f_a(\omega_7) = f_a(\omega_8) = t_a^2$

From a state space to a type space

t_a^2	1/4	1/4	<i>H</i>	0	1/2
t_a^1	1/4	1/4	<i>L</i>	0	1/2

1/3	1/4	1/3	1/4
<i>H</i>	<i>H</i>	<i>L</i>	<i>L</i>
2/3	3/4	2/3	3/4
t_b^1	t_b^2	t_b^3	t_b^4

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From a state space to a type space

t_a^2	1/4	1/4	<i>H</i>	0	1/2
t_a^1	1/4	1/4	<i>L</i>	0	1/2

1/3	1/4	1/3	1/4
<i>H</i>	<i>H</i>	<i>L</i>	<i>L</i>
2/3	3/4	2/3	3/4
t_b^1	t_b^2	t_b^3	t_b^4

- $f_a(\omega_1) = f_a(\omega_2) = f_a(\omega_3) = f_a(\omega_4) = t_a^1$
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- $b_a(t_a^1) = (\frac{1}{4} \otimes (H, t_b^1); \frac{1}{4} \otimes (H, t_b^2); \frac{1}{2} \otimes (L, t_b^4))$
 $b_a(t_a^2) = (\frac{1}{4} \otimes (H, t_b^1); \frac{1}{4} \otimes (H, t_b^2); \frac{1}{2} \otimes (L, t_b^4))$

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Proof.

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Questions???